

# **An Amateur Vertical Component Broadband Seismometer (Model MkXX)**

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## **ABSTRACT**

A vertical component broadband (BB) seismometer, based on design ideas from numerous sources, was developed and fabricated by an amateur instrument builder in a home workshop using commonly available materials. The seismometer had a flat velocity response between 0.017-4 Hz, and sensitive enough to detect teleseismic events of magnitude 5.5 and greater from most places on earth.

## **INTRODUCTION**

For teleseismic event detection, the popular amateur choice is a long period horizontal component seismometer fitted with an electromagnetic transducer to produce an output signal proportional to ground velocity. Unfortunately, the electromagnetic transducer's output signal gradually gets weaker towards the longer periods and a horizontal component is very susceptible to ground tilting. To improve sensitivity at the lower frequencies, the MkXX had a displacement transducer whose output signal was proportional to mass displacement. A vertical component (Z-axis) was selected for the instrument, for better immunity to ground tilting. The displacement transducer's output signal was modified and fed back to a second transducer, a voice coil, mounted on the pendulum to provide a broadband output signal flat to velocity, improve system linearity and increase bandwidth.

The vertical component design required the added complexity of a spring suspension system to counterbalance the force of gravity acting on the inertial seismic mass. Excessive pendulum drift, due to the temperature sensitivity of the carbon steel suspension spring, required a small DC electric motor to periodically restore transducer equilibrium. To protect the suspended mass from atmospheric pressure fluctuations, the entire mechanism had to be packaged in a rigid airtight housing.

Complexity of the mechanical design was limited by the equipment and materials readily available from local hardware stores and metal salvage yards. A range of powered hand tools, a mill/drill machine, metal cutting band saw and a 3" metal lathe (hobby type) assisted with the cutting and shaping of the thicker materials. To save time and materials, several of the mechanical components were salvaged from previous homemade seismometers. Aluminum, not normally chosen by professional instrument designers [1] for serious structural components, was readily available and easily machined.

Because the seismometer was built in a non-metric country, linear dimensions are specified in either inches or millimeters. The purpose of this document was to describe briefly how the MkXX was designed and made, with no intention of laying out a set of step by step instructions for duplicating a second instrument.

## **MECHANICAL DESIGN**

### **INSTRUMENT ENCLOSURE & FRAME**

The backbone of the enclosure was an aluminum tube 6.00" (152.4 mm) OD x 12.00" (305 mm) long x .125" (3.12 mm) wall thickness, see Figures 1 and 2. Though the cylindrical shape complicated the design of the seismometer, its circular cross section may have provided more rigidity, than a flat plate, for supporting the internal mechanism. A flat plate of insufficient thickness will buckle, when differential air pressure changes act on opposing sides of the plate. Discs of 1/8" thick opaque Plexiglas covered the ends of the tube. One disc was bonded in place and the other was made removable, secured to the tube with six 1/4-20 UNC metal studs bonded to the OD of the tube. Short lengths of brass wires 1/32" dia passed through the removable end plate. These brass wires were sealed to the end plate with epoxy after the signal carrying wires (and their shields) were soldered together. Rubberized cork gasket material 1/16" thick sheet was cut to size and heavily coated on both sides with a layer of petroleum jelly (Vaseline brand), for an airtight seal between tube and end

plate. The grease soaked into the cork, requiring a second application two days later. Hand tightened knurled nuts and washers secured the plate to the tube assembly.

Longitudinal and lateral movement of the frame inside the tube was controlled by pieces of aluminum sheet metal bonded to the inside of the tube. The forward piece was “V” notched to receive the front internal leveling foot hanging below the frame’s base plate, and two rear pieces had holes to engage the two other internal leveling screws, which protruded at a shallow angle at the rear of the frame. Bonded to the under side of the tube were two pieces of square aluminum tubing reinforced on the ends with solid pieces of hexagon bar stock, to carry three ¼-20 UNC coach head bolts used for leveling the instrument. On the top of the tube was a lifting handle screwed and sealed in place, and a small circular spirit level bonded adjacent to it.

The frame of the internal mechanism could not be fastened directly to the tube, but rested on three feet that contacted the inside surface of the tube. Parts deliberately had thick cross sections wherever possible to add mass, thermal inertia and rigidity. The enclosure was leveled prior to installing the mechanism. Next the leveling screws were adjusted on the internal frame, to level it correctly relative to the enclosure. Note: a separate plate (black colored in several photos) provided a temporary way of supporting the frame while working on it externally from the enclosure, having three screws of its own

## PENDULUM AND SUSPENSION SPRING

Wielandt and Streckeisen [2] pointed out that it is preferable to achieve high acceleration resolution by maximizing the pendulum’s period where possible, thereby placing less demands on the displacement transducer’s sensitivity. A couple of old patented designs for vertically suspended pendulums with natural periods longer than 1 second were considered for the MkXX. First was the diagonally mounted LaCoste “zero length” spring [3] which can theoretically provide an infinite period, but the spring is susceptible to lateral low frequency vibrations, causing “bumps” in the data. Second was Willmore’s patent [4] from 40 years ago. Its cantilevered tapered leaf spring was relatively simple and more resilient to lateral forces, with a natural frequency way above the pass band of interest. Obtaining a copy of Willmore’s patent is highly recommended, for the detail information regarding the leaf spring. Only those aspects of the spring’s design which influenced the MkXX will be discussed (briefly) below.

The first figure in Willmore’s patent, i.e. *FIG 1*, is reproduced in this paper as Figure 10 because it identifies many important relationships and proportions which must be observed for the suspension to function successfully. Additional text has been placed on the diagram for the reader’s convenience. Essentially, the pendulum is in its equilibrium position when a pre-bent tapered cantilevered leaf spring is straightened under load and inclined at a 45 degree angle. The lower end of the spring is clamped rigidly to a structure in common with the pendulum’s hinge (flexures). Over a limited distance, centered about pendulum’s equilibrium position, deflection of the actual spring’s free end closely follows the path of an “ideal” spring, i.e. one that provides the pendulum with an infinite period over a much longer distance. A tie (flexible link) of constant length connects the pendulum to the free end of the spring. Theoretical starting values on the MkXX for D1 and D2 were both 71mm, and 100mm for distance H2/Q, the active portion of the spring.

To create the curved tapered spring, the handle and reinforcement bar was first removed from an 8” wide blue steel tapping knife blade, measuring 0.0185” thick. With the blade clamped to the edge of the bench, a Dremel tool fitted with a Dremel #409 cut-off wheel was used to cut out a trapezium shaped part measuring 50mm across the bottom, 6mm across the top and 130 mm high. Any burrs (very little found) were removed using a grinder. The lower end of the spring was inserted to a depth of 25mm between aluminum clamp plates and the top end of the spring lost 5mm to a smaller clamp, leaving a length of 100mm for the spring to freely flex. Due to spring back, the flat spring was curved by manually over bending it around a piece of 1.00” OD PVC tube clamped in the bench vice until it assumed a radius roughly equal to half the active length of the spring (equal to a 50mm radius). A paint can, close to the required radius was used to gage the final curved shape. After installation and under load, the spring did not want to fully straighten itself out for last 13mm below the upper clamp, see Figure 4. Only made the one spring, and was used “as is” for this experiment.

Carbon steel’s spring modulus varied in proportion to the ambient temperature. Warmth weakened the spring and cool air stiffened it, causing the equilibrium of the pendulum to alter enough to require compensation. So the clamp at the base of the spring was pivoted on two 1/8” diameter dowel pins, manipulated by a small 12-volt DC motor coupled to a 1000:1 ratio gearbox fitted with a worm which rotated a Nylon gear on a steel rod threaded into a brass swivel nut at the end of an arm attached to the spring clamp. By rotating the clamp about the dowel pins on command, more or less force was exerted by the spring, keeping the pendulum in equilibrium. Spring and motor assembly was its own module, seen to the right in Figure 4 attached to the base plate, also shown in Figure 5. Base plate was slotted for adjusting the position of the spring relative to the pendulum’s hinge.

Pendulum and spring were coupled together with a (adjustable) link, identified in Figure 10 as the TIE. At each end of the tie were pieces of phosphor bronze foil 0.002” thick clamped to the tip of the spring and to a cross bar on the pendulum. Exit points for the foils from beneath the clamps were initially set up at 71mm away from the hinge. Brass wire links soldered to the foils and linked together with a small turn-buckle with wire hooks. The turn-buckle was just another method for later adjustability.

Two pieces of 0.002" thick by 0.50" wide feeler strip formed the hinge (flexures), attaching the 150mm long pendulum to a pedestal on the base plate. Over a quarter kilogram of brass was added to the side pendulum's side plates for seismic mass, including a 44 gram trim weight seen screwed to the top edge of a side plate. Mounted between, and to, the side plates were the moving vented outer plates of the displacement transducer and a nylon rod supporting the feedback transducer coil. Non-magnetic materials had to be used on the pendulum, because the transducer's magnet was attached to the frame nearby. No method for locking the pendulum was provided. If the seismometer had to be carried, it was tilted back at an angle, causing the pendulum to lift until the displacement transducer's plates rested together.

Final note: when the curved spring was first subjected to the weight of the pendulum, some initial stress relieving occurred. After releasing the spring from the tie and was allowed to relax (in the curled position), the tip fell short of returning to its original location by ~10mm. This step was performed several times, and the spring returned to its new relaxed position every time with no further changes in shape. Nothing could be done immediately to correct it. A higher than expected frequency of automatic transducer re-centering cycles would have been an indication that the spring could not consistently maintain its position under a constant load.

## **FEEDBACK TRANSDUCER**

A voice coil, working in conjunction with a permanent magnet elevated above the base plate, Figure 8, applied a feedback force proportional to ground acceleration that both modified and damped the pendulum's motion. Transducer linearity directly effects the accuracy of the final output signal.

Sean-Thomas Morrissey utilized the coil/magnet assembly from a large 12" off-the-shelf audio speaker as a feedback transducer on his unique STM-8 design [5]. Standard audio speakers cannot control very much mass because of their low generator constant (the term  $G_n$  used in later feedback calculations), so a custom-made feedback transducer was necessary for the MkXX. Read Sean-Thomas Morrissey's e-mail postings archived on the Public Seismic Network (PSN) web site for instructions pertaining to their design and how he made custom transducers.

Figure 8 partially shows some of transducer's mechanical detail. The magnet was salvaged from an 8" mid-range speaker. Its voice coil gap was opened out a little to accommodate a slightly beefier voice coil. Cleaned the gap in the magnet very thoroughly by sliding pieces of vinyl insulation tape through it. By forcing the sticky side of the tape against the inside surfaces of the gap, it captured loose magnetic particles, flakes of paint, lint, hair and dead skin. A large piece of tape was temporarily placed over the exposed poles and gap to prevent dirt and metallic particles from re-entering. Tape was removed at final assembly with the coil.

The voice coil bobbin was made from several wraps of 0.001" thick plastic film bonded together and to a plastic end plate 1/8" thick containing 4 vent holes. These items were supported by a metal plug (wrapped in some Teflon plumbers tape to prevent the bobbin from sticking to the plug) until the adhesive was cured, at which time the plug with tape was removed. A length of 36 gauge enameled magnet wire was wrapped around, and bonded to, the bobbin. Coil resistance equaled 87 ohms. Bobbin sat on top of a non-eddy current forming nylon rod.

## **DISPLACEMENT TRANSDUCER**

Desiring good performance from the homemade instrument, a full bridge, or symmetric differential capacitive (SDC) transducer [6] was selected. Compared to a variable capacitance half bridge design, an SDC transducer has double the sensitivity, a well defined null position and does not require a very precise symmetric oscillator. Capacitive transducers are generally quieter electronically and more sensitive than a linear voltage displacement transformer (LVDT) transducer. Its does however require two variable capacitors and a precise method for differentiating their output signals. SDC transducers basically consists of two separate displacement transducers coupled together to move as a single unit, with each transducer forming a branch of the bridge circuit. When responding to detectable motion between the seismic mass and base plate, each transducer's output was of opposite polarity to the other, providing a true differential output signal.

The three-plate transducer design balanced out the electrostatic forces acting between the plates. Viscous air trapped between the plates posed a significant problem by damping the pendulum's motion, so a pattern of vent holes were uniformly distributed across all plates of the transducer. Size of gaps between the plates and vent hole diameters had to be carefully considered for an acceptable sensitivity. Critical aspects of (half bridge) capacitance transducer designs have been covered in a paper by Jones & Richards [1] and a book written by Baxter [7], but the information is still applicable to the full bridge SDC design.

Fastened to the pendulum were the two outer plates (modulated signal transmitters), each one divided electrically into two separate active areas, facing towards the center plate. Each of the outer plate's active areas was joined electrically to the diagonally opposite active area on the opposing outer plate. The center plate (modulated signal receiver), mounted to the base plate, was also divided electrically into two active areas. Whenever one of the outer plates moved closer to the center plate, one half of the center plate received a stronger positive signal while the other half of the center plate received a stronger negative signal. Fine 40 AWG coax wire carried the modulating signal from the oscillator circuit to the outer plates. These coaxes easily flexed where they bridged the gap at the pendulum's hinge. The outer plate's active areas

overlapped the sides of the center plate, allowing for some very slight unavoidable off-axis motions without noticeably effecting sensitivity.

The center capacitor plate was made from 1/8" thick FR-4 (PCB) material coated on both sides with 1 oz, 0.0014" thick, copper foil. Outer plates were 1/16" thick coated on one side with 1 oz, 0.0014" thick copper. It was assumed that the board's manufacturer probably applied some type of corrosion inhibitor on the exposed copper. Boards were cut to shape with a saw and a Dremel Tool engraved isolation grooves 1-2 mm wide, to define the active areas in the copper layers. Mounting screws and metal spacers were electrically isolated from the active areas. Outer braided jackets of the modulating signal's coaxes were only tied to the electronics' metal enclosure to extend the shield. The actual active area for each of the two separate displacement transducers was calculated to be 979 square millimeters, which excluded the vent holes areas. Nominal gaps either side of the center plate measured 1.0 mm. Parallelism between the center and outer plates were within 0.08 mm at the extremes of pendulum travel. Air viscosity was less of an issue when the total vent hole area was greater than 15% of the total plate area (i.e. active area + vent hole area) for the given nominal gaps. Sensitivity was not adversely compromised.

## **ADJUSTING THE MECHANICAL FREE PERIOD**

Before commencing this part of the build process, all mechanical items had to be assembled together. Both transducers and re-centering motor were fully wired through to the access plate, but not connected to the circuit boards. Then the internal mechanism was placed on the temporary support stand and leveled.

Referring again to Figure 10, spring length H2/Q and tie length H1/H2 started out at 100mm, and distances D1 and D2 were 71mm apiece. Seismic and trim masses were altered regarding their size and locations, until the pendulum was horizontally balanced by the spring. Surprisingly, the pendulum's natural period exceeded 3 seconds before any adjustments had to made! Altering the geometry by increasing dimension D2 to 74mm and shortening the tie's length to 97.5 mm established a natural period of 4.4 seconds (the goal was 4 - 5). Imperfections in the spring/tie assembly required the spring tip to move slightly forward from the ideal position directly above the hinge, see Figure 4. The pendulum oscillated freely, permitting the natural period to be averaged over 10 cycles, and to verify that air trapped between the capacitor plates and around the voice coil was not a serious concern.

## **ELECTRONICS**

The electronic circuits were modularized according to their unique function within the overall system, making it possible to conveniently swap out a board for upgrade or repair. There were four separate hand-wired boards, the Oscillator Circuit Board (OCB), Displacement Transducer Circuit Board (DTCB), Feedback Circuit Board (FCB) and the Transducer Re-centering Circuit Board (TRCB). The OCB and the DTCB were mounted together in the same aluminum project box. Likewise, the FCB and the TRCB shared space in their own aluminum box.

Power came to the circuit boards from an off-the-shelf +/-12 VDC dual linear supply (1 amp per side). Most of the op amps were reasonably priced for their rated performance, less than a dollar each, but some were around \$5 each. Approximately 100 mA of current was drawn from either supply. Activation of the re-centering motor drew an additional 50 mA from either supply.

The electronic circuits were constructed using metal film resistors with 1% tolerances, for all values up to 1 meg ohm. Larger values were carbon types with 5% tolerances. All power decoupling caps and feedback cap values <0.01 microfarad were ceramic, all others >0.01 mF were polyester. The circuits were built on perforated boards with sockets for all ICs. To reduce power rail noise, the power pins on all op amps were wired via 20-Ohm resistors to a common point on the output pins of the voltage regulators. To eliminate or significantly reduce ground loops, a small piece of adhesive backed copper foil 3 mm wide by 20 mm long was attached to the underside of the perf board, between the rows of legs of each op amp socket. Decoupling caps and other components associated with each op amp requiring grounding were soldered to the piece of foil. Then each individual foil was connected directly to a common ground point on the circuit board, no daisy chaining. Also, .01 mF ceramic decoupling caps were inserted where power-carrying wires entered a circuit board.

A short length of coax cable connected the output of the DTCB to the FCB. The coax outer shield was connected to each metal project box. Any flux residues seen built up on the circuit boards, extending between conductors, were scraped off as best as possible to reduce potential signal leakage.

## **OSCILLATOR CIRCUIT BOARD (OCB)**

Fixed voltage regulators designated U1 and U2 (see Figure 11) filtered and stepped down the voltage from the common supply, providing +/-5 VDC. The input and output capacitors filtered out most of the voltage spikes on the supply rails. The IN4002 diodes protected the regulators from excess voltages during power down.

Figure 11 shows a 1 MHz crystal oscillator U3 followed by two TTL divide-by-ten counters U4 & U5, to obtain a frequency of 10 KHz with a 0 to +5 V square wave output. A NPN transistor Q1 with a bipolar output converted it into a +/- 5 volt square wave signal. A voltage divider prevented the buffer U6 from being saturated. A 3<sup>rd</sup> order low pass filter with a corner frequency of 5 KHz reduced the square wave to its fundamental frequency by removing its higher order harmonics. Op amp U7 added gain to restore some of the signal lost going through the filter, pumping out a ~ 2 V P-P sine wave signal.

### **DISPLACEMENT TRANSDUCER CIRCUIT BOARD (DTCB)**

The DTCB was mounted inside the same metal enclosure that housed the OCB, since both boards had high frequency signals. Adjustable voltage regulators U8 and U9, Figure 12, filtered and stepped down the voltage from the common supply, providing the circuits with +/-10 VDC. With 10 mF Tantalum capacitors connected to the regulators, better than 65dB ripple and spike rejection was possible. The IN4002 diodes protected the regulators from excess voltages during power down.

AC coupled, inverting op amp U10, in Figure 13, boosted the incoming sine wave signal to ~ 10 V P-P. An audio transformer T1 (Digi-Key P/N 237-1121-ND) had 600-600 ohm center tapped coils which were referenced to ground for DC common-mode voltage rejection. The secondary coil produced two modulating signals, 180 degrees out of phase with each other and connected to the active areas on the SDC's outer plates, PA and PB, per the diagonally opposing pattern described earlier in the paper and shown in Figure 13.

Op amp U11 is a zero-crossing detector referenced to ground for converting the AC sine wave into a sharp square wave, oscillating from 0 to +10 volts. Because the demodulator's analog switch U14 required a lower triggering voltage, a divider dropped its peak to +3.8 volts. The active areas on the capacitor's moving center plates, PC and PD detected the modulated voltage signal's amplitude and phase, in proportion to its displacement in either direction from their null position. Op amps U12 and U13, both high impedance follower amplifiers at unity gain, drove guards (the coax shields) to reduce stray input capacitance for additional sensitivity. The input pins to these op amps were bootstrapped for circuit stability. Their outputs were fed into the analog switch U14, a demodulator synchronized with the sine wave signal from U10, monitoring the modulated voltage of each half cycle. Op amp U15 is an instrumentation amplifier (in-amp) with 51X gain, differentiating the weak signal from the SDC. Coax audio cables, half a meter long, were soldered to the SDC and connected to the circuit board via RCA phono plugs. The outer shields of the two outer plate's coaxes were attached to chassis ground.

### **FEEDBACK CIRCUIT BOARD (FCB)**

Adjustable voltage regulators U8 and U9 (Figure 12), including their associated components were duplicated on this board to supply filtered +/- 10 VDC to the circuit. Referring now to Figure 14, op amp U16 is configured as a special integrator with the intended purpose of improving low frequency sensitivity and stability. Such an "integrator" appeared in papers written by Erhard Wielandt [2][8][9], placed adjacent to the displacement transducer in block diagrams representing the electronic circuitry of Streckeisen seismometers. Erhard Wielandt, via personal communications, explained that because the system would oscillate if high gain is maintained at high frequencies, loop gain had to be frequency-dependent, increasing towards the low frequencies while ensuring unity gain at the higher frequencies. One way to realize this is to insert a special integrator or quasi-integrator, within the feedback loop. Based on the word description he provided and after some experimentation, a workable circuit design evolved. A 1<sup>st</sup> order 34 Hz LPF attenuated high frequency circuit noise from the displacement transducer prior to entering the quasi-integrator. Op amp 16 is a unity gain instrumentation amplifier (IA) with high impedance FET inputs and excellent common mode rejection (CMR) to differentiate between small input signals. A 2<sup>nd</sup> order 3.5 Hz high pass filter (HPF) comprising of C2, C3, R2 and R3 was inserted into the integrator's feedback loop between pins 2 and 6 of the IA. Note: sensitivity to local high frequency seismic events suffered because of the low HPF corner frequency could have been raised it to 10 Hz. Buffer op amp U17 installed after the integrator prevented any loading of the downstream circuits. A temporary shunt was installed across the capacitors C2 and C3 when the system was calibrated and removed for normal operation.

The multi-path feedback circuit followed a schematic published by Sean-Thomas Morrissey for his vertical component VBB seismometer, the STM-8 [5] which was itself based on similar circuits found on Streckeisen STS-1 & STS-2 VBB seismometers. Multi-path feedback is a combination of differential and integral feedback signals flowing through the feedback transducer (voice coil) mounted on the pendulum. The differential signal dominates across the passband of interest, decreasing towards the lower frequencies where the strength of the integral signal increases. Reading the papers by Morrissey and Wielandt is highly recommended for their discussions on this subject. Resistor Rp and capacitor C provided the proportional feedback. The combined currents of Rp, C and RI flowed through the feedback coils, modifying the pendulum motion for a broad band velocity response. The capacitor designated C, was a single Mylar cap.

Following the quasi-integrator, op amp U18 inverted the acceleration signal before it entered a composite integrator, comprising of U19 and U20. The RC time constant of the 10 Meg resistor and the 10-microFarad capacitor (at 100 seconds) is the mathematical term TI used in later calculations. A composite integrator [10] takes advantage of the high

input impedance of a FET op amp combined with the DC output accuracy of a BJT op amp. Op amp U21 buffered the circuit whenever the displacement signal was monitored with a DVM. The TRCB also monitored the displacement signal at pin 6 on op amp U20.

Buffer U22 prevented down stream loading of a .003 Hz (333 seconds) high pass filter to remove unwanted DC signal drift. Op amp U23 with 2X gain drove a capacitive load created by a 30-meter long coax cable connecting the seismometer to the data logger, a PC fitted with an internal A/D card.

## **TRANSDUCER RE-CENTERING CIRCUIT BOARD (TRCB)**

Whenever the displacement transducer's signal exceeded +/-1.30 volts of offset after a long period of integration, the motor was activated. Long period integration eliminated false triggering during the detection of large microseisms, wind noise and seismic events. The pendulum was returned to null at a rate slightly slower than the lagging response of the feedback circuit's integrator output signal (observed when the seismometer is correctly set up). Complementary power boosting transistors were configured to form a dead band within plus or minus 0.6 volts of null. When the amplified signal of the transducer reached the dead band on either side of null, the output voltage of the transistors quickly dropped off to a level insufficient to drive the motor. Then the feedback circuit's lagging integrator output could catch up and drift into the dead band, placing the pendulum very close to null without dithering or overshoot. Automatic (remote) re-centering meant that the thermal insulation surrounding the instrument was not disturbed.

The TRCB was mounted inside the same metal enclosure that housed the FCB, sharing the FCB's two voltage regulators that were a duplication of U8 and U9. This circuit constantly monitored the voltage level and polarity of the displacement output at pin 6 on op amp U20.

Referring to Figure 15, op amps U24A and U24B buffered the circuits from the effects of the Long Period Integrator (LPI), formed by the 2 Meg resistor and 2X 1500 mF caps. The LPI's time constant of 1500 seconds, or 25 minutes, was enough to smooth out voltage spikes that would have constantly triggered the motor into action. Op amps U25A and U25B are voltage comparators to output >8 volts whenever the LPI exceeded voltage levels of +/-1.3 volts. Their high state outputs turned on transistor Q2, driving the input of a LM555 timer U26 low. For the next 11 seconds, the output of U26 remained high, causing transistor Q3 to activate a DPDT relay. One pole of the relay, REL 1A, drained accumulated voltage away from the large capacitors to re-set the LPI. A momentary push button switch SW1 mounted on the metal enclosure was used at any time to manually override the function of the LPI, by driving the input of U26 low. As a visual indicator, an LED mounted on the outside of the metal enclosure lit up whenever the re-centering circuit was activated. Like U24A, op amp U27 constantly monitored the voltage and polarity at pin 6 on U20. Power transistors Q4 and Q5 boosted the current output of op amp U27 to drive the DC motor at over 50 mA. Once activated, the relay pole REL 1B connected the motor, MOT, indirectly to the feedback circuit. By being able to track the circuit's polarity, the motor rotated in a direction necessary to null the displacement transducer. The 30K-feedback resistor increased the motor drive voltage for optimum speed. At the end of the 11-second timed cycle, the motor was once more isolated from the rest of the circuit.

## **BROADBAND VELOCITY OUTPUT SIGNAL**

The accepted standard is to have a positive going voltage output signal streaming from the vertical component seismometer whenever the ground motion is upward. To test circuit polarity, the pendulum was gently pushed down (equivalent to the ground moving upward) to verify that a positive voltage signal was present at pin 6 on U16, U20, U21 or U23. A negative voltage signal was in fact discovered, requiring the wires to pins 1 and 2 on U14 to be reversed. Powering up the circuits caused the pendulum to violently oscillate, until the wires to the feedback coils were reversed. Not shown in the electronic schematics was a simple 1:2 voltage divider, installed between the output of U23 and the A/D card, thus preventing it from receiving a voltage signal exceeding its rated input of +/- 5 volts.

# **TRANSFER FUNCTION**

## **DISPLACEMENT TRANSDUCER CONSTANT: V/M**

Jumper wires must first be temporarily installed across capacitors C1 & C2 to avoid the filter's influence at DC. The transducer's displacement constant Volts/Meter (V/M), was determined by measuring the voltage at pin 6 of U16, at TP1, as the pendulum was moved in 0.0005" (0.0127 mm) increments about its null position. A micrometer's spindle contacted the pendulum at the seismic mass' center of gravity, calculated to be 131mm from the hinge. The collected voltage/displacement data points were then plotted on graph paper. Looking at the plotted data points, transducer linearity exceeded 85% of the range of travel, in either direction from its null position. Precisely pushed the pendulum for a distance of 0.004" (0.1016 mm) roughly centered about the null position, well within the linear portion of the plotted points. Over this distance, the SDC transducer produced a combined total output of 13.09 volts, which equaled 128,838 V/M. Note:

additional amplification of the seismic signal in later circuits saturated the final output well before the linear range of the transducer was exceeded. The gain resistor value on op amp U15 was adjusted for the desired sensitivity. Both jumper wires were removed after performing this calibration step.

### **GENERATOR CONSTANT: $G_n$**

This test set-up applied a small vertical force to the pendulum. First, the mechanism was leveled and the transducer roughly nulled. Temporarily disconnected the feedback coil from the electronic circuit board and connected a DVM to pin 6 of op amp U15. Connected a 1.5 V battery, an on/off momentary switch, a mA meter (another DVM) and a 100K potentiometer in series with the feedback coil. Placed a small test weight of known mass at the center of the seismic mass (131mm from the hinge), heavy enough to deflect the pendulum downwards and bottom out the displacement transducer. Now the battery circuit was turned on and the 100K potentiometer adjusted until just enough current flowed through the voice coil to null the displacement transducer, indicated by ~0 volts on the DVM connected to pin 6 of U15. Did not allow more than a few milliamps to flow through the coil, otherwise the coil could have burned out. The amount of current necessary to null the transducer was recorded with the test weight resting on the pendulum.

The test mass was a paper clip measuring 0.0004331 Kg. Re-centering current was measured as 0.00019 Amp. The generator constant,  $G_n$ , is defined in units of Newton / Amperes (N/A).  
 $N = \text{mass} \times \text{acceleration} = 0.0004331 \times 9.8 = 0.004244 \text{ N}$   
 $G_n = N/A = 0.004244 / 0.00019 = 22.3 \text{ N/A}$

### **SYSTEM RESPONSE**

The transfer function is a mathematical statement that describes the relationship between the input, a set of predetermined parameters, and the output of the system. Calculating a system's output can be a very difficult task. Fortunately the MathCAD file compiled by Morrissey for his STM-8 was available for downloading [11], thereby saving a great deal of time and effort. MathCAD Standard 2000 software was used to open the file on a local PC. The STM-8's values were substituted with different ones, see Appendix A at the end of this paper (the MathCAD work sheet). Numerous values were inserted for C,  $R_p$ ,  $R_I$  and  $T_I$ , before settling on those below for an acceptable response:

#### Input parameters:

$r = 129,000 \text{ V/m}$  (this value was determined per the method explained above)  
 $M = 0.43 \text{ Kg}$   
 $C = 0.000010 \text{ Farads}$   
 $R_p = 800 \text{ K Ohms}$   
 $R_I = 82 \text{ K Ohms}$   
 $G_n = 22 \text{ N/A}$  (this value was determined per the method explained above)  
 $R_f = 87 \text{ Ohms}$   
 $T_I = 100 \text{ seconds}$   
 $T_o = 4.4 \text{ seconds}$

#### Calculated system response:

$Zeta = 0.71$  (damping)  
 $T_n = 57 \text{ seconds}$  (closed loop natural period)  
 $V/m/s = 1,957$  (max sensitivity)

The plotted curve for the velocity response showed the high frequency end rolling off at roughly 10 Hz (see Appendix A), yet it was the quasi-integrator that limited the top end to about 4 Hz.

## **SEISMOMETER VAULT**

Vault co-ordinates: Latitude: 47.849 N, Longitude: 122.328 W, Elevation: 110 meters

The seismometer was installed in a small surface vault squeezed between a 1.5-meter tall hedge and a 2-meter high wooden fence in the backyard of a house located in a noisy urban environment. Vault was separated from bedrock by a 1.8 meter thick layer of unconsolidated backfill on top of 200+ meters of unconsolidated glacial till (according to a geologic map). Long period noise mostly occurred when winds struck the hedge and fence. High levels of short period noise came from the railroad tracks located one kilometer from the vault, where on an average day twenty to thirty freight trains passed by. The Pacific Ocean, at a distance of 160 kilometers to the west, generated microseisms with periods of 4 - 6 and 15 seconds.

Inside the vault was a small rectangular pier, measuring 450mm wide x 800mm long x 250mm thick, cast from a 50/50 mixture of sand and cement (no aggregate) poured directly onto compacted earth. Styrofoam sheet, 75mm thick, isolated the pier from the vault's, 203mm thick, hollow concrete block walls. An insulated plywood roof was hinged

off one of the block walls to permit access to the vault's interior, from the top. Styrofoam insulation 25mm thick lined the internal wall surfaces and 50mm thick sheets covered the external surfaces of the concrete block walls.

Used the spirit level bonded to the top of the enclosure to roughly level the seismometer. Under each seismometer leveling screw was a piece of smooth glass plate 40mm x 40mm x 5mm thick, for reducing thermal noise and torsion forces necessary to operate the screws. The seismometer was powered up, with a DVM connected to pin 6 on U21. Then a leveling screw was adjusted to tilt the instrument forward or backward, until a reading of 0.00 +/- 1.00 volt was displayed. This was followed by toggling the momentary switch SW1, for precise nulling using the re-centering motor.

Installed over the instrument (excluding electronics) was a Styrofoam cover. Internal dimensions of the cover were approximately 10-20 mm greater than the maximum width, length and height of the seismometer. The intent was to stratify the air temperature trapped inside the enclosure to minimize convection currents. Five rectangular shaped pieces of 25mm thick sheet were hot melt glued together. Seams were sealed with a second bead of adhesive to ensure all gaps were filled. Cable bundle from the seismometer passed through a snug-fitting notch cut in the lower edge of the cover at one end, where it contacted the pier. After the cover was installed on the seismometer (and without letting the cover touch the instrument), two layers of fluffy 90 mm thick fiberglass insulation were added.

Never during normal operation was the pendulum truly nulled, it always showed some amount of offset. An internal (instrument) temperature change of roughly 1.7 degrees C triggered re-centering, recognized in the data as a pair of large amplitude waves, slowly diminishing back to normal behavior within 6 to 8 minutes. An automatic re-centering cycle restored optimum performance, sometimes occurring once a day or once a week (depending on ambient conditions). Lots of insulation helped to control this situation.

## DATA PROCESSING

Collected data through an 8 channel 16 bit A/D card (model PSN-ADC-12/16 Rev 2 from Larry Cochrane of Redwood City, California USA) installed unshielded on the motherboard of a NEC 233 MHz Pentium desktop PC. The A/D card's clock crystal was housed in a temperature-controlled oven and corrected by a WWV time signal. Clock drift was accurate enough for this amateur application. One channel recorded the raw output signal (0.017-5 Hz passband, defined by the seismometer's internal filters) for detecting high frequency (short period) regional events, and a second channel recorded data passband filtered at 0.003-0.05 Hz for detecting long period teleseismic events.

Data was displayed and saved using Seismic Data Recorder (SDR) software, Version 4.1, collected at a rate of 50 samples per second. Seismic events files were time and date stamped by the software re-sampled and saved in PSN format. WinQuake software, Version 2.9.8, analyzed the data files created with SDR. Adjustable digital filters further controlled band pass limits and roll-offs, for extracting useful seismic data from the clutter of unwanted background noise. After entering the event's origin time and co-ordinates into WinQuake, the embedded JB Tables calculated the theoretical arrival times for P, S and other wave phases, placing markers at the proper locations on the seismograms.

## RESULTS

How well did this instrument detect events? See several sample seismograms, saved as GIF images using WinQuake software, starting at Figure 16, on page 20. Under each event is an accompanying caption containing preliminary event information. Viewing this document as a PDF image may require zooming in on parts of the traces to reveal finer details. Event files were routinely re-sampled at a lower rate (5 - 15 Hz) to reduce file size.

Without the appropriate test equipment and skills necessary to assess the instrument's sensitivity and noise levels, visual comparisons were made with near real time seismograms accessible on the WWW. Best candidate for the study was station COR (USGS) located 360 km to the south in Corvallis Oregon, where a Streckeisen STS-1 VBB Z-axis seismometer (VBB response flat to velocity between 0.003-5 Hz) provided the data. The previous 24 hours of data was displayed passband filtered at 0.01-0.05 Hz (long period), along with high gain. Each day, numerous small teleseismic events were usually present on its seismograms. During favorable local weather conditions, the MkXX successfully detected many of the same events using narrow digitally defined pass bands. Background noise levels observed on the MkXX during an average day were of course higher than COR. It was difficult to compare data from an instrument supported by unconsolidated earth, to one at a remote site some distance away residing on solid bedrock.

## CONCLUSIONS

Four to five weekends were spent to design, build and calibrate the seismometer. Complexity of the motorized suspension and a air-tight housing, will deter most amateurs from trying to replicate the design. It was not cheap either by amateur standards, costs of the raw materials and electronic components totaled approximately \$250.

Willmore's suspension was a practical solution for the MkXX, and should work just as well on a vertical seismometer fitted with a conventional electromagnetic transducer, even if the period is only 5 - 10 seconds. On that type



of instrument, spring creep could be compensated with a screw device, similar to one described in the patent. Cutting the tapered leaf spring to size was not very difficult, but cautiousness and patience paid off.

Mathcad calculations had determined that a low end corner response of 120 seconds was feasible. Operating nearer to 50 seconds proved to be more compatible with the prevailing vault conditions i.e. thermal noise, while still providing an acceptable long period performance. Successful local event detection requires raising the quasi-integrator's rather low top end response to something above 3.5 Hz. For a velocity response of 1 Hz and much higher, a vertical component electromagnetic transducer may be more suitable, that's if a second instrument can be tolerated. The original goal was to optimize the MkXX for teleseismic event detection.

Environmental noise was the most dominating in the data, followed by instrument noise. Naturally, during stable atmospheric conditions the lowest long period background noise levels were observed. Overall, when considering the price and time taken to build the seismometer, the results were very satisfying, in spite of its crude appearance.

## ACKNOWLEDGEMENTS

The author is indebted to the late Sean-Thomas Morrissey who for several years generously shared with subscribers to the Public Seismic Network (PSN), his professional knowledge and experiences regarding seismometer theory, operation and installation, particularly the construction details of the STM-8. Erhard Wielandt must be thanked for his mentoring and explanations regarding the quasi-integrator.

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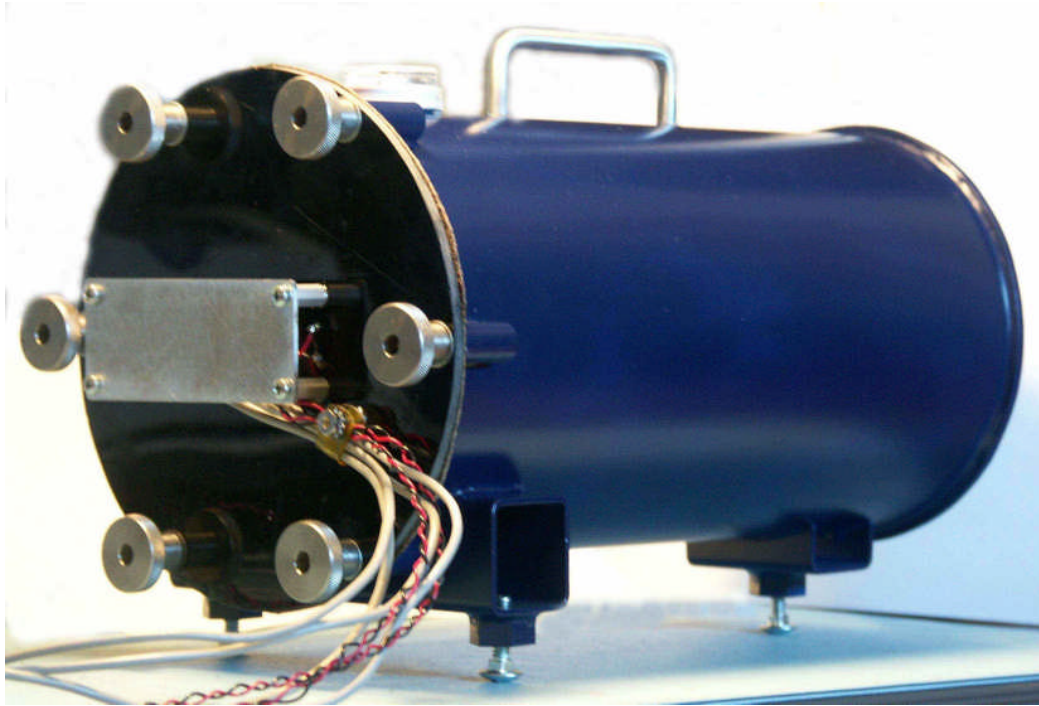
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[http://www.geophys.uni-stuttgart.de/seismometry/man\\_html/index.html](http://www.geophys.uni-stuttgart.de/seismometry/man_html/index.html)
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[http://mnw.eas.slu.edu/People/STMorrissey/etc\\_export](http://mnw.eas.slu.edu/People/STMorrissey/etc_export)

## SUGGESTED READING

Usher M. J., Buckner I. W. & Burch R. F., 1977. *A miniature wideband horizontal-component feedback seismometer*. Journal of Physics E: Scientific Instruments, **10**, 1253-1260.

Usher M. J. & Guralp C., 1978. *The design of miniature wideband seismometers*. Journal of Geophysics, **48**, 281-292.

## FIGURES



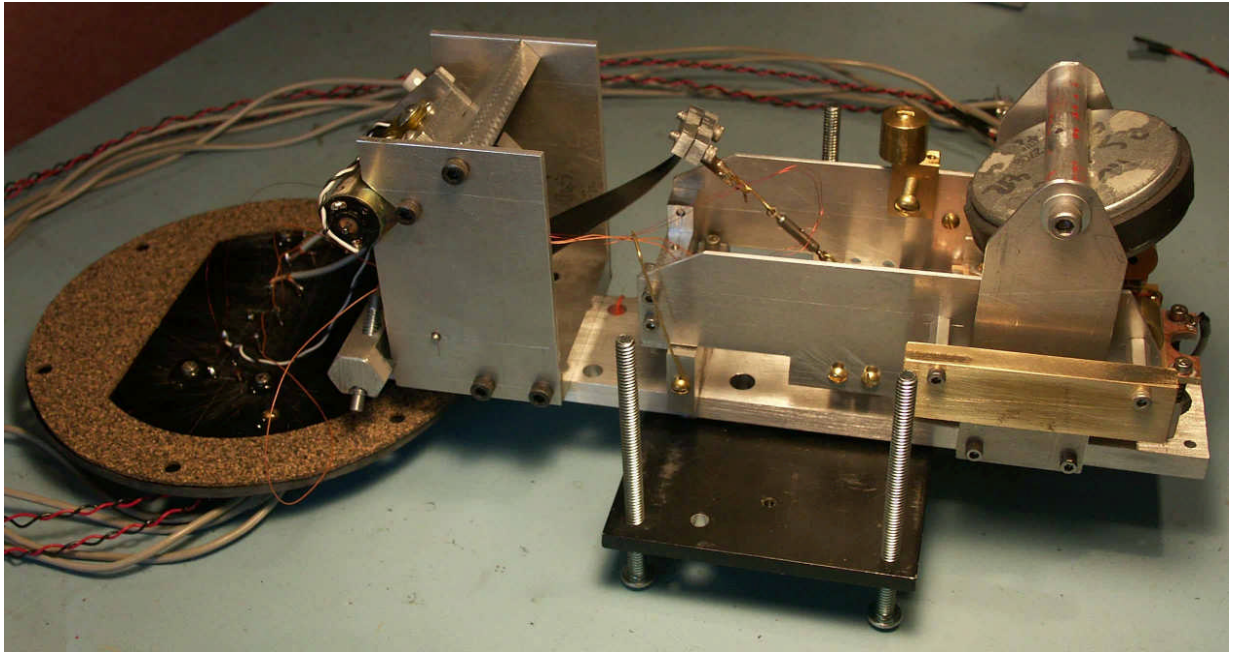
**FIGURE 1**

The final design: showing the cable bundle attached to the removable end plate (black disc) held in place with 6 knurled knobs. Rectangular aluminum plate protects wire terminations. Unit rests on three adjustable feet.



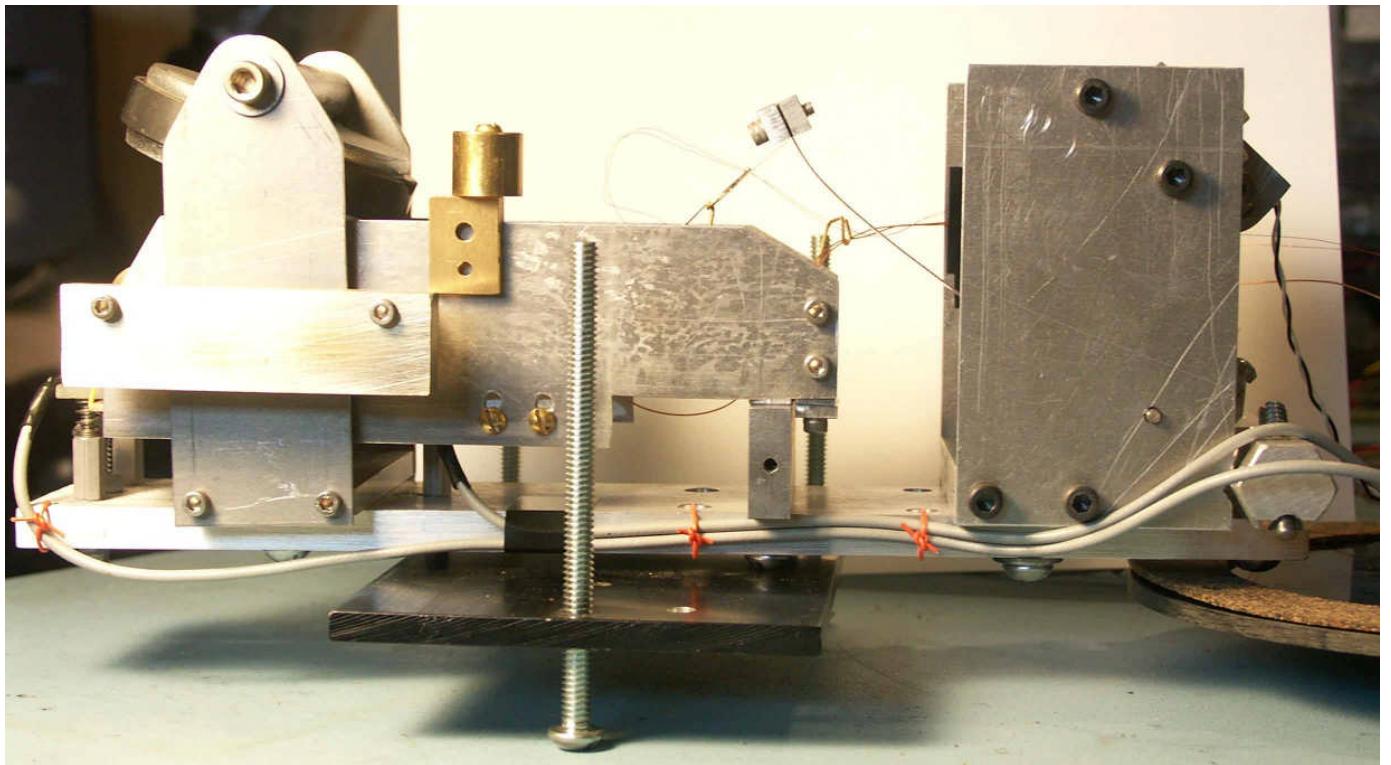
**FIGURE 2**

End plate removed to expose the inside of the tube. Three small plates, bonded to the inside surface, receive the leveling screws on the internal mechanism. Front plate (far end) has a "V" notch and the back two (near end) have holes. Lifting handle and spirit level fixed to top of the tube.



**FIGURE 3**

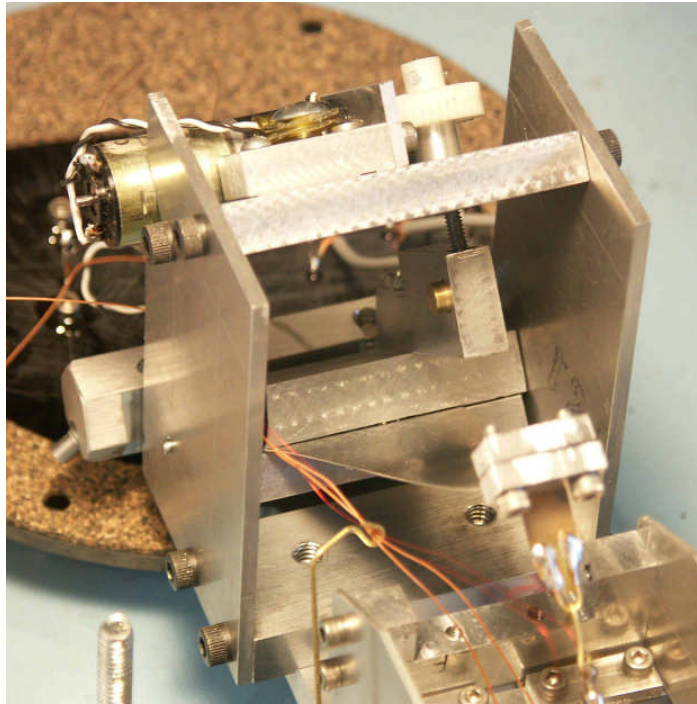
Overall view of the internal mechanism, seen resting on the (black colored) temporary support stand. Features visible from left to right: plastic end plate with a cork gasket, leaf spring protrudes from its motorized module with a link, inclined at a 45° angle, connected to the horizontal pendulum, speaker magnet inclined at an angle above pendulum.



**FIGURE 4**

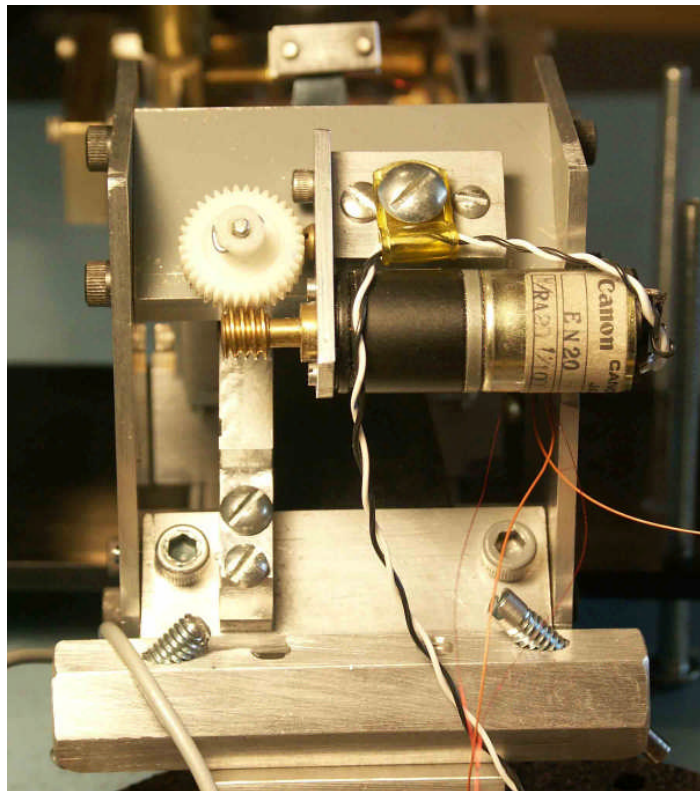
Side view of the internal mechanism: Leaf spring seen on edge, protruding from the upright assembly to the right. Link joined to spring tip above the pendulum's hinge. An adjustable brass trim weight rests on the pendulum's side plate. The heavy gray wires along the base plate are coaxes to the center plate of the displacement transducer.





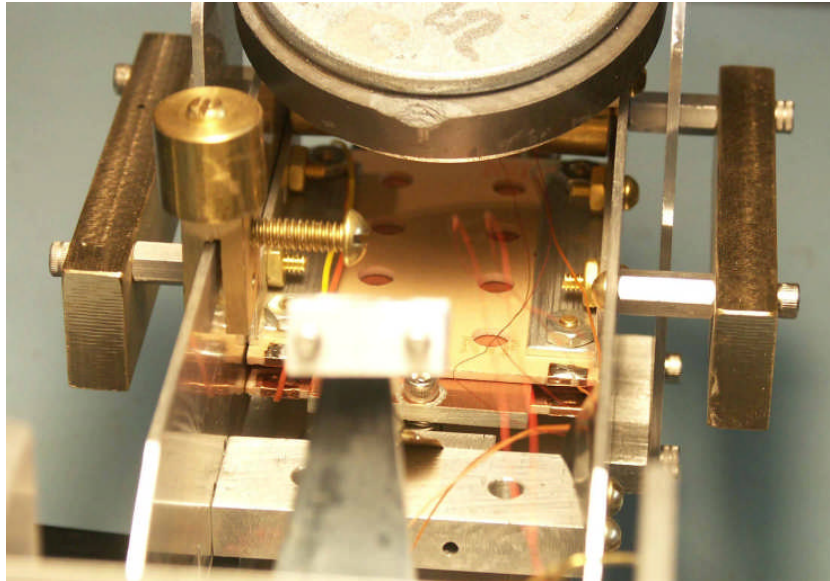
**FIGURE 5**

Tapered leaf spring is clamped to a motorized bar which pivots on two dowel pins. One pin is shown protruding from the near center of the vertical side plate.



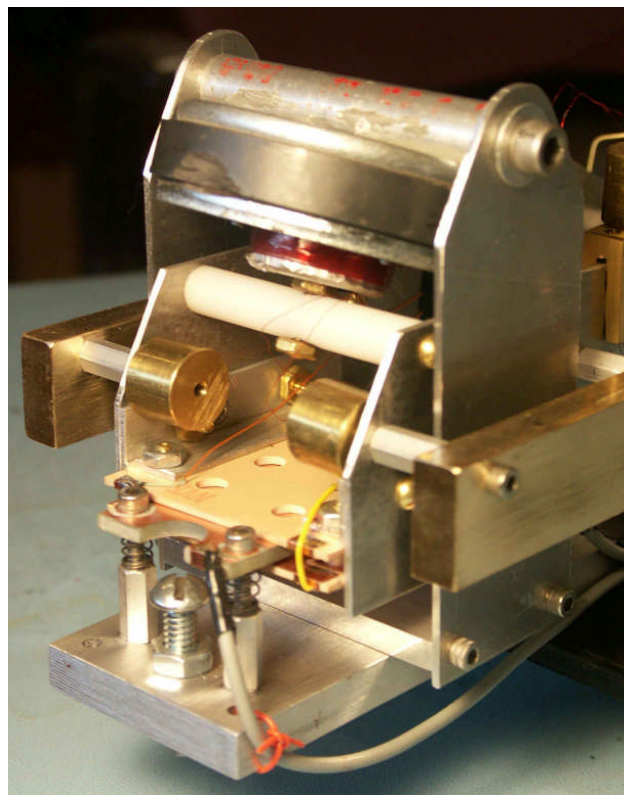
**FIGURE 6**

A DC motor, through reduction gearing, manipulates the base of the tapered leaf spring. At the bottom of the photo are two angled pins which eventually engage the small plates bonded to the inside of the tube.



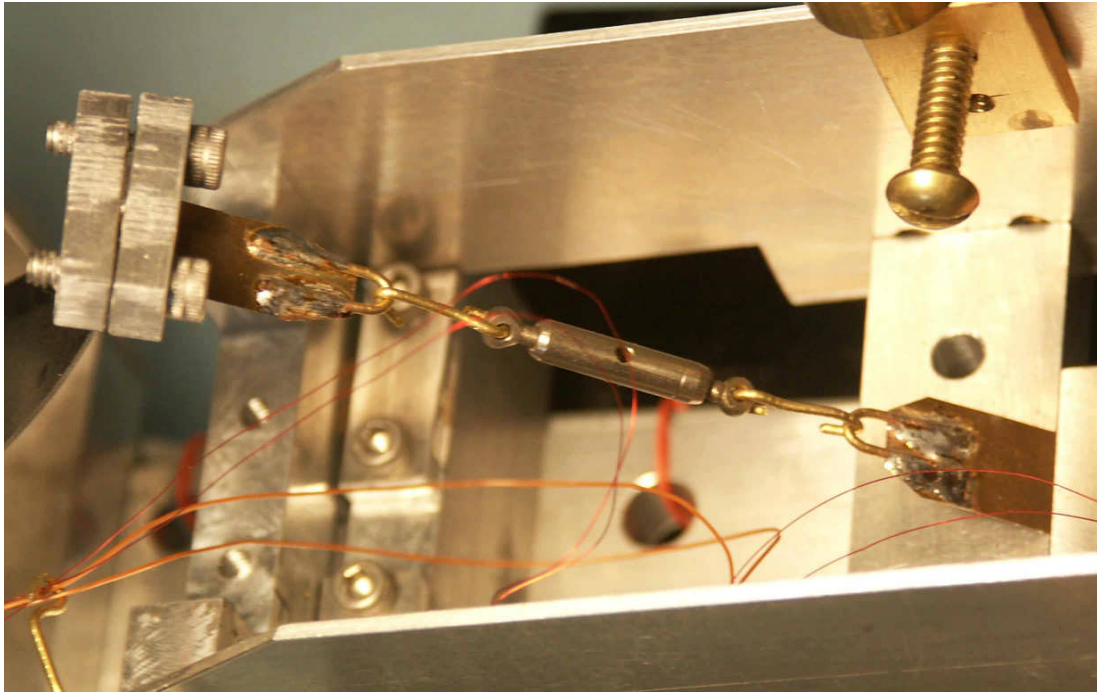
**FIGURE 7**

Looking down on the pendulum assembly, the tip of the leaf spring is in the foreground. Yellow colored plate with the symmetrical hole pattern, beneath the speaker magnet, is the moving upper plate of the displacement transducer. Large brass side bars are part of the seismic mass.



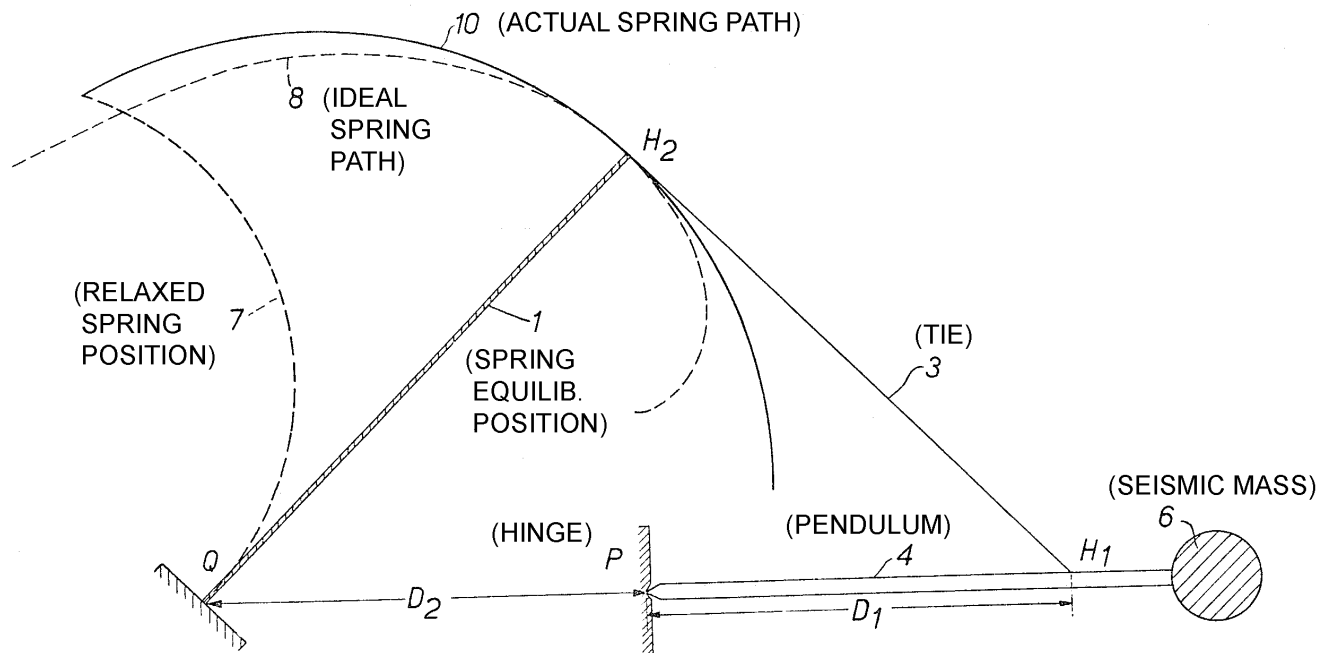
**FIGURE 8**

End view of the internal mechanism: Voice coil, mounted on white nylon rod, is seen entering the speaker magnet. The displacement transducer's center plate is supported in three places (2 shown) with springs and screws for convenient adjustability. The larger vertical screw seen locked with a nut near the end of the aluminum base plate (below notch in transducer center plate), is one of the three screws for leveling the internal mechanism.



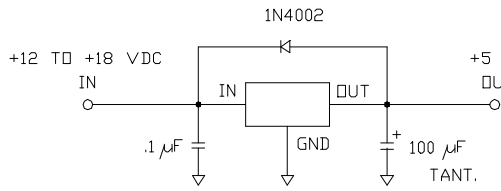
**FIGURE 9**

View of the adjustable tie between spring tip and pendulum cross bar: Brass wire links are soldered to flexures, with a miniature turn-buckle in between for adjustability. The orange colored wires are the fine coaxes carrying the sine wave signals to the moving plates of the displacement transducer. The fine red wires lead to the voice coil.

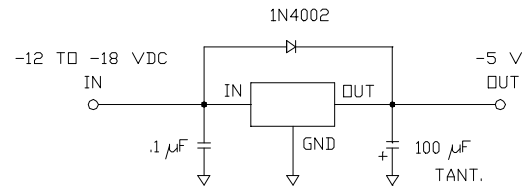


**FIGURE 10**

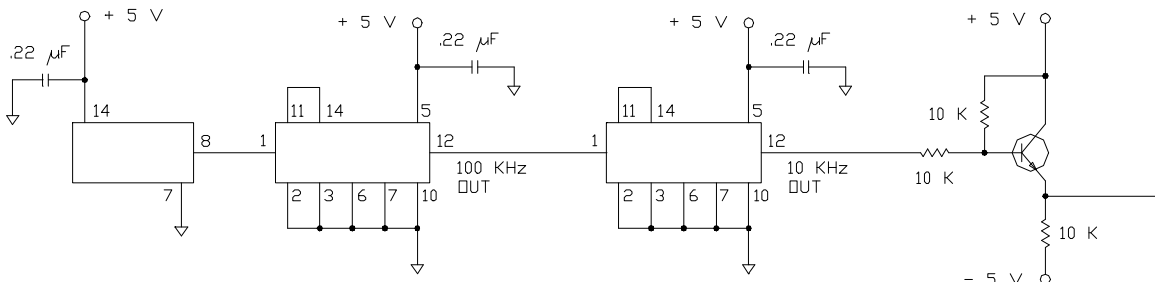
Included in the paper for quick reference, is the diagram which appeared as *FIG 1* in Willmore's patent. Text has been added to the key design features, to help explain the concepts. Numbers and letters relate to descriptive text found in the patent.



U1  
LM7805



U2  
LM7905

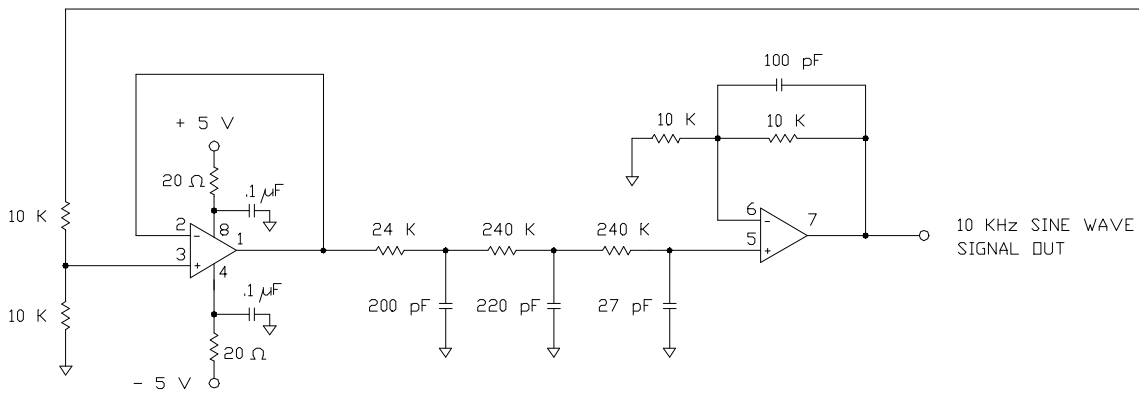


U3  
CLOCK OSCILLATOR  
F1100E 1 MHz

U4  
74LS90

U5  
74LS90

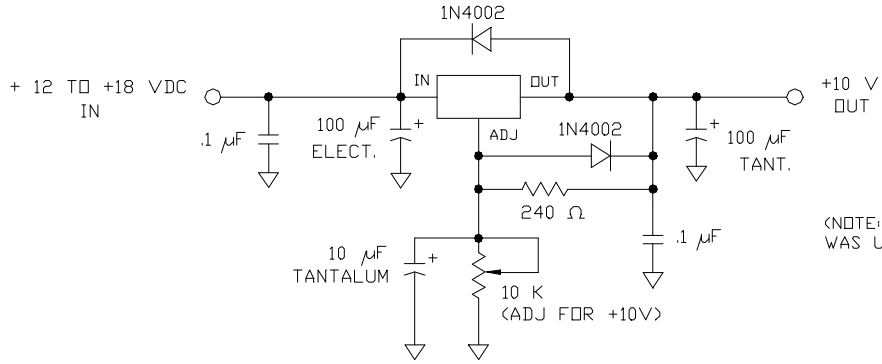
Q1  
2N2222



U6  
LF412

U7  
LF412

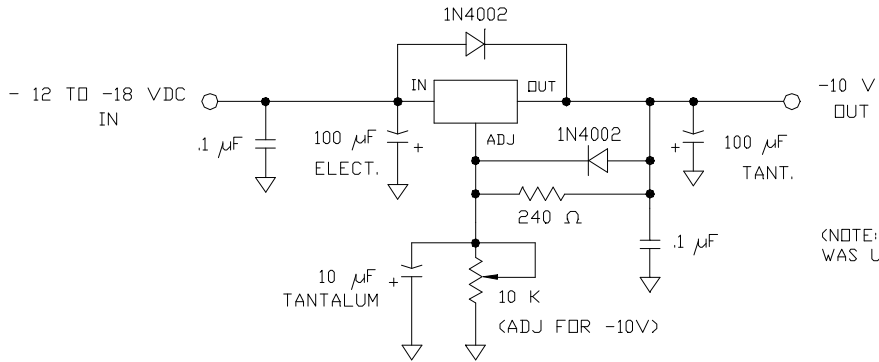
FIGURE 11  
OSCILLATOR CIRCUIT BOARD (OCB)  
REVISED 25 JUNE 2005



<NOTE: THIS VOLTAGE REGULATOR WAS USED ON THE DTCB AND FCB>

U8

LM317



<NOTE: THIS VOLTAGE REGULATOR WAS USED ON THE DTCB AND FCB>

U9

LM337

CONNECT TO GND ON  
COMMON POWER SUPPLY

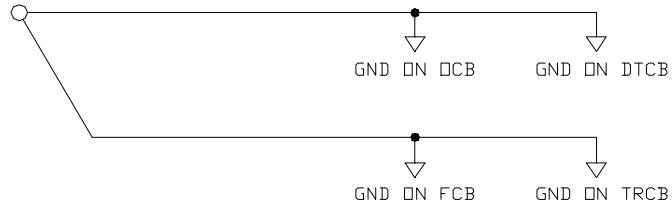
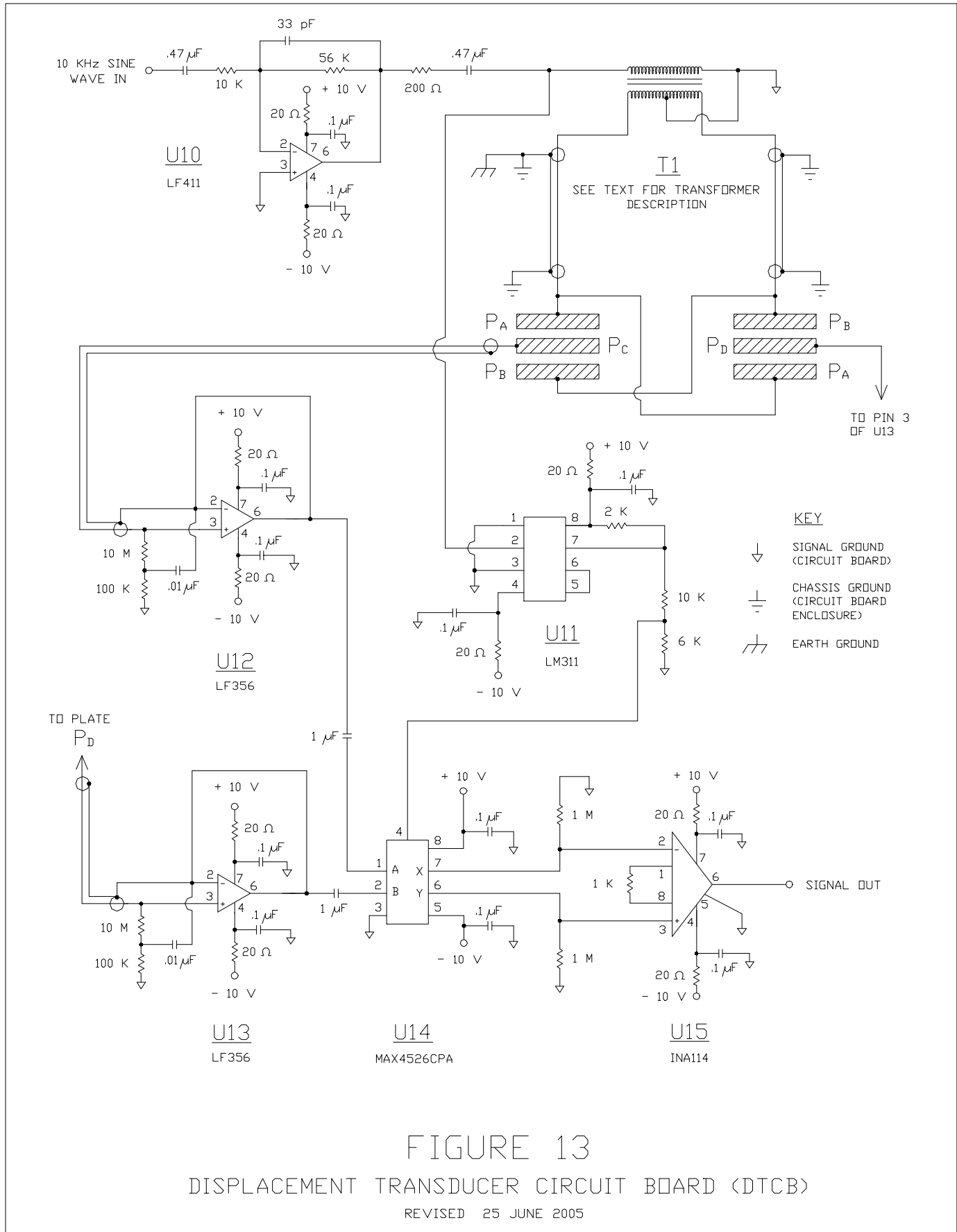


FIGURE 12  
VOLTAGE REGULATOR CIRCUITS

REVISED 25 JUNE 2005





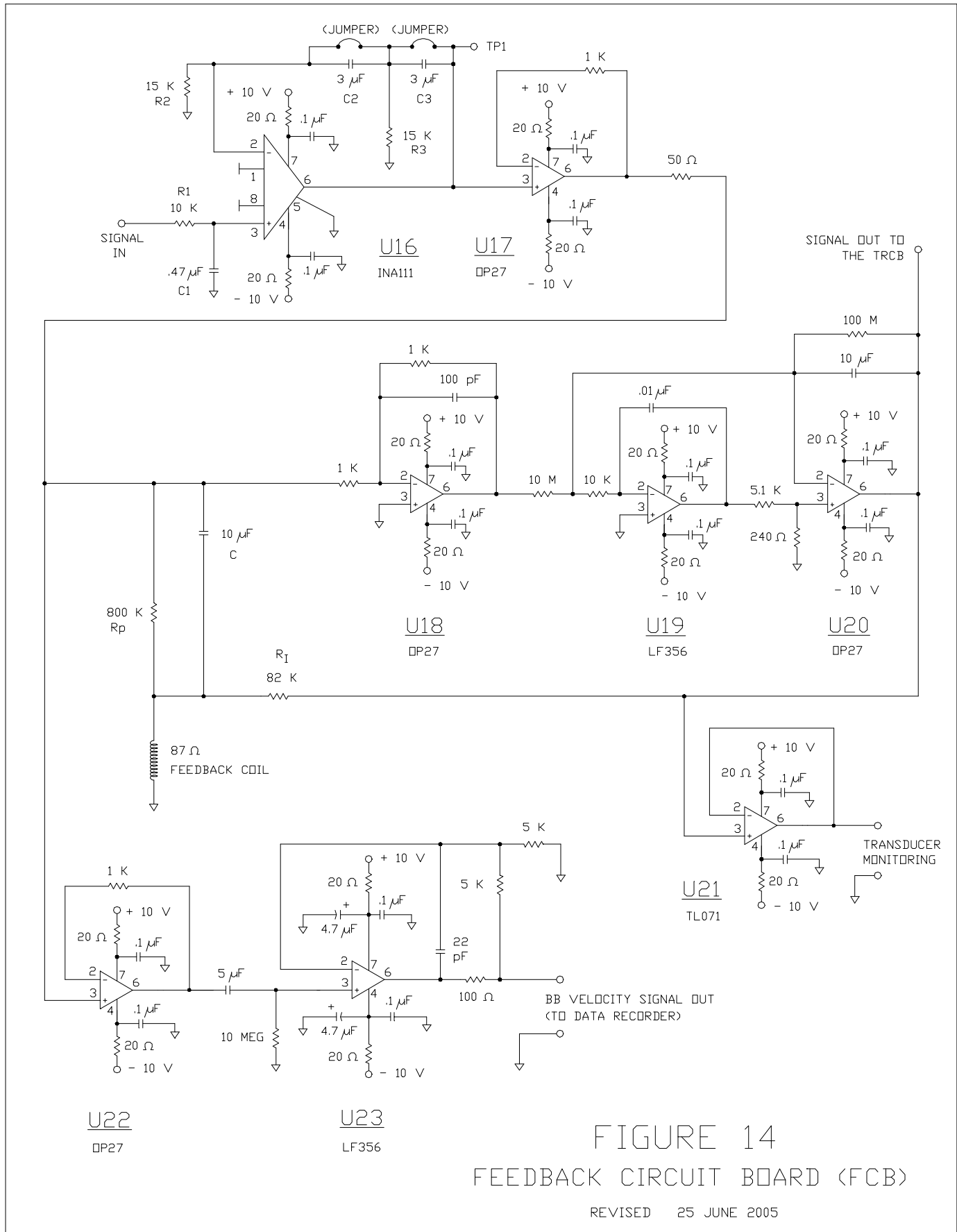


FIGURE 14  
 FEEDBACK CIRCUIT BOARD (FCB)

REVISED 25 JUNE 2005

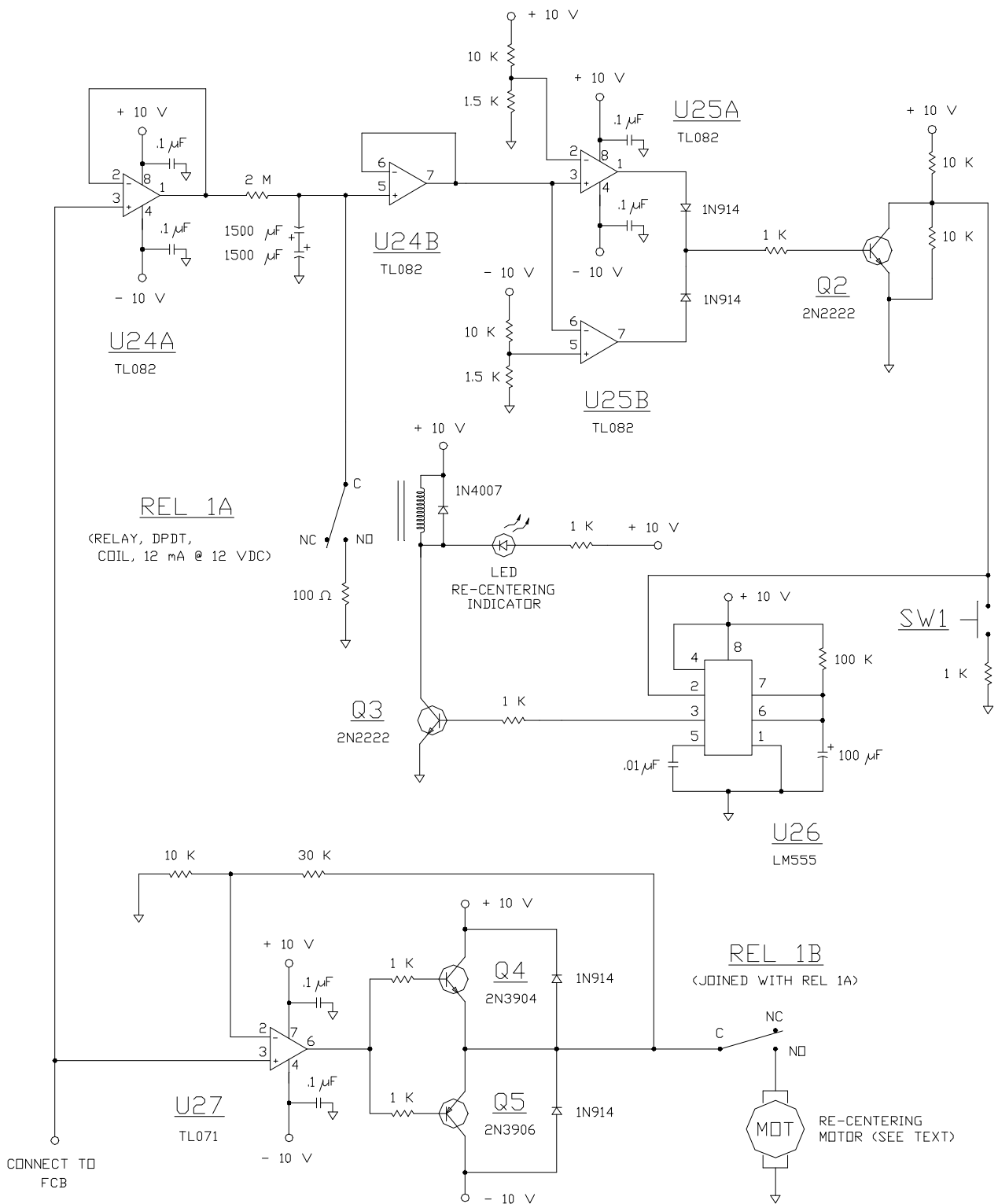
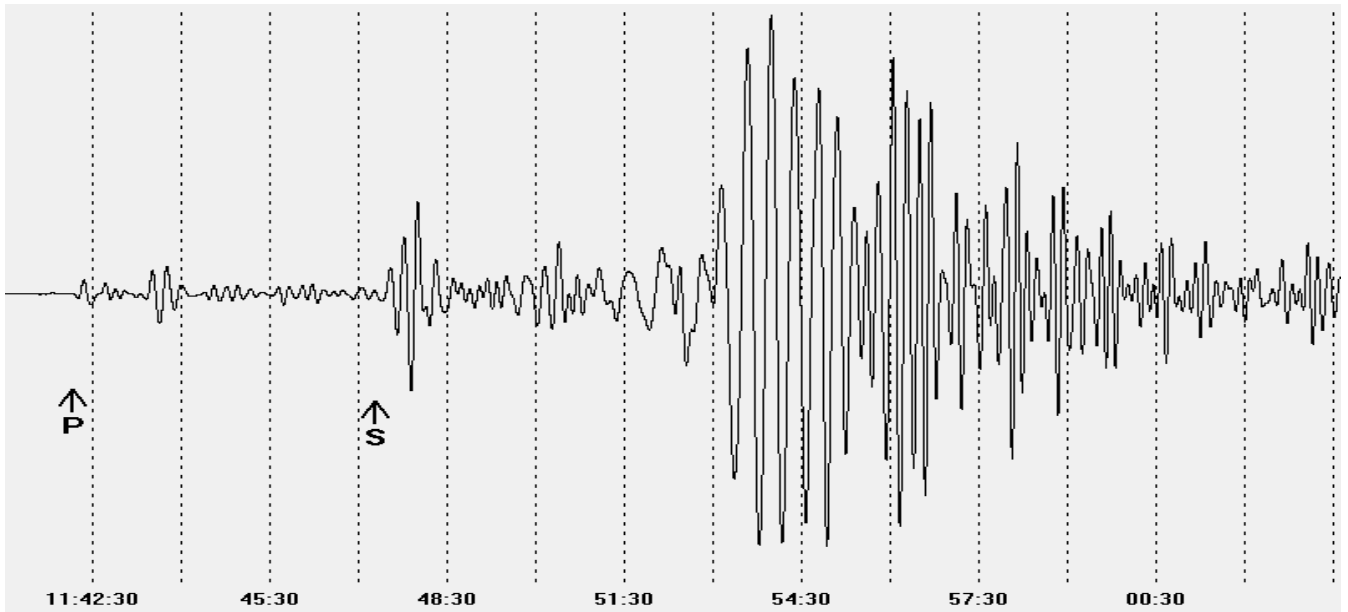


FIGURE 15  
 TRANSDUCER RE-CENTERING CIRCUIT BOARD (TRCB)

REVISED 25 JUNE 2005



**FIGURE 16**

SEISMOGRAM: M6.2 - OFF THE COAST OF JALISCO, MEXICO

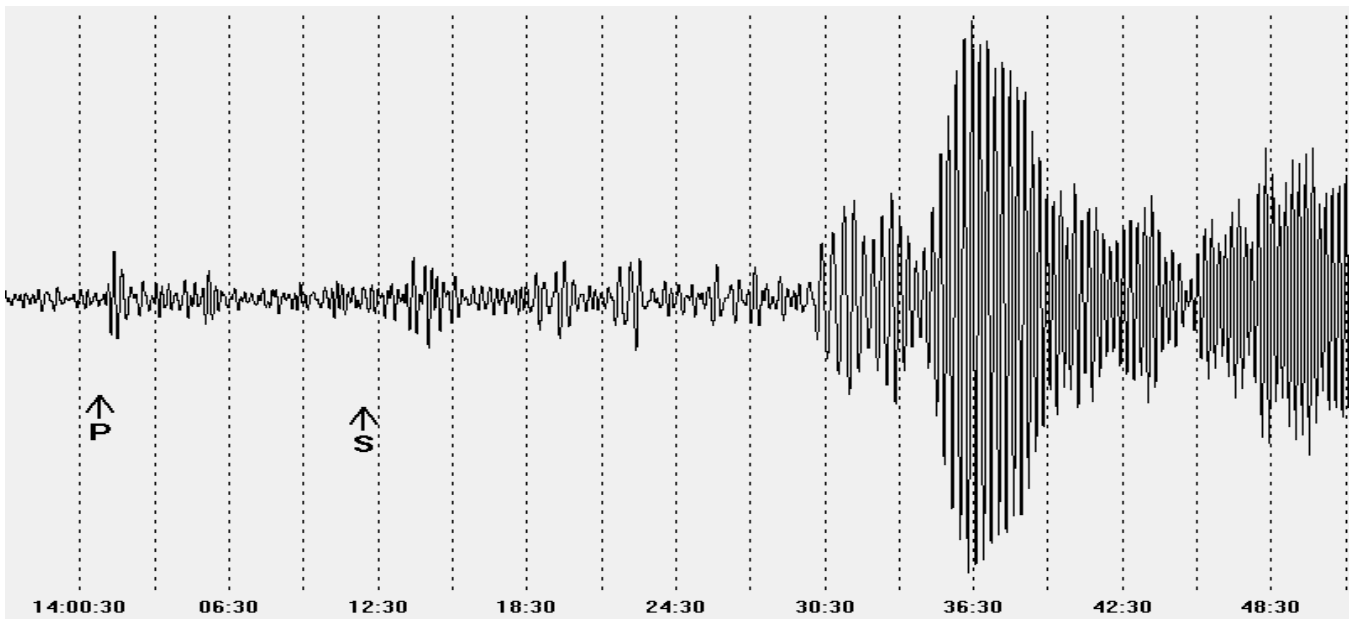
Origin date and time: 2005/06/27, 11:35:44 UTC

Event coordinates: 18.77° N, 107.31° W, 10 km deep

Delta = 31° (3,486 km)

Passband: 0.02-0.10 Hz.

UTC time indicated across the bottom of the seismogram. Vertical grid spaced at 1.5-minute intervals.



**FIGURE 17**

SEISMOGRAM: M5.6 - SOLOMON ISLANDS

Origin date and time: 2005/06/30, 13:48:29 UTC

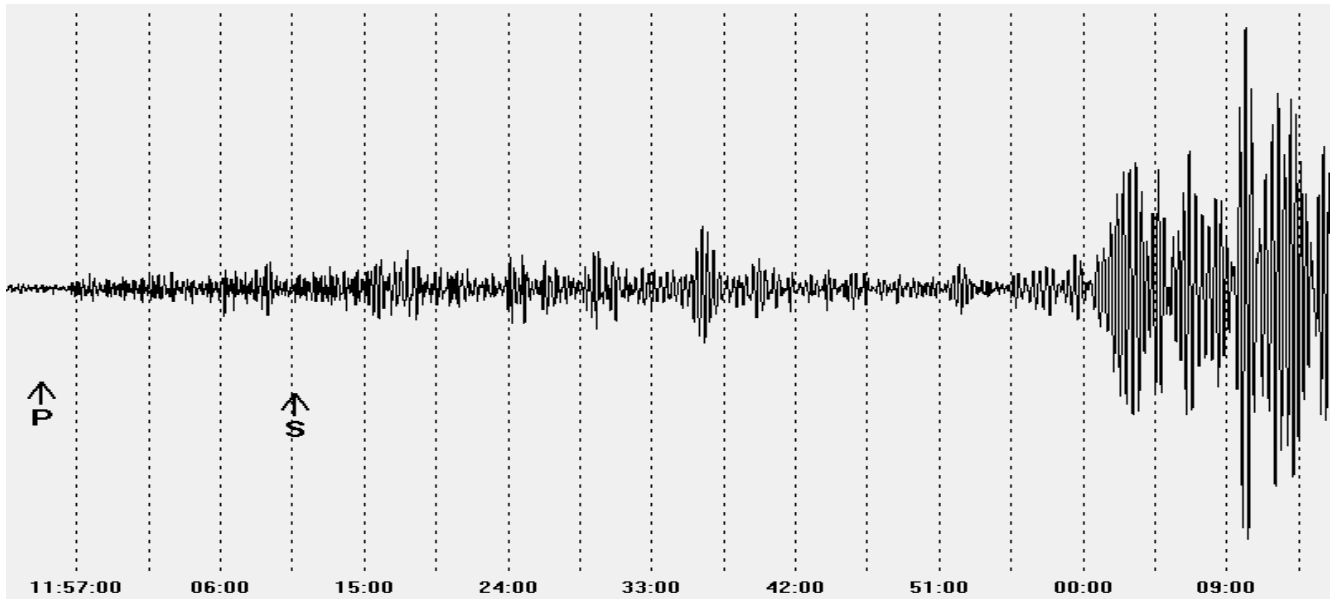
Event coordinates: 10.90° S, 162.34° W, 65.7 km deep

Delta = 88° (9,823 km)

Passband: 0.035-0.08 Hz

UTC time indicated across the bottom of the seismogram.

Vertical grid spaced at 3-minute intervals.



**FIGURE 18**

**SEISMOGRAM: M6.3 - PRINCE EDWARD ISLANDS REGION**

Origin date and time: 2005/07/04, 11:36:00 UTC

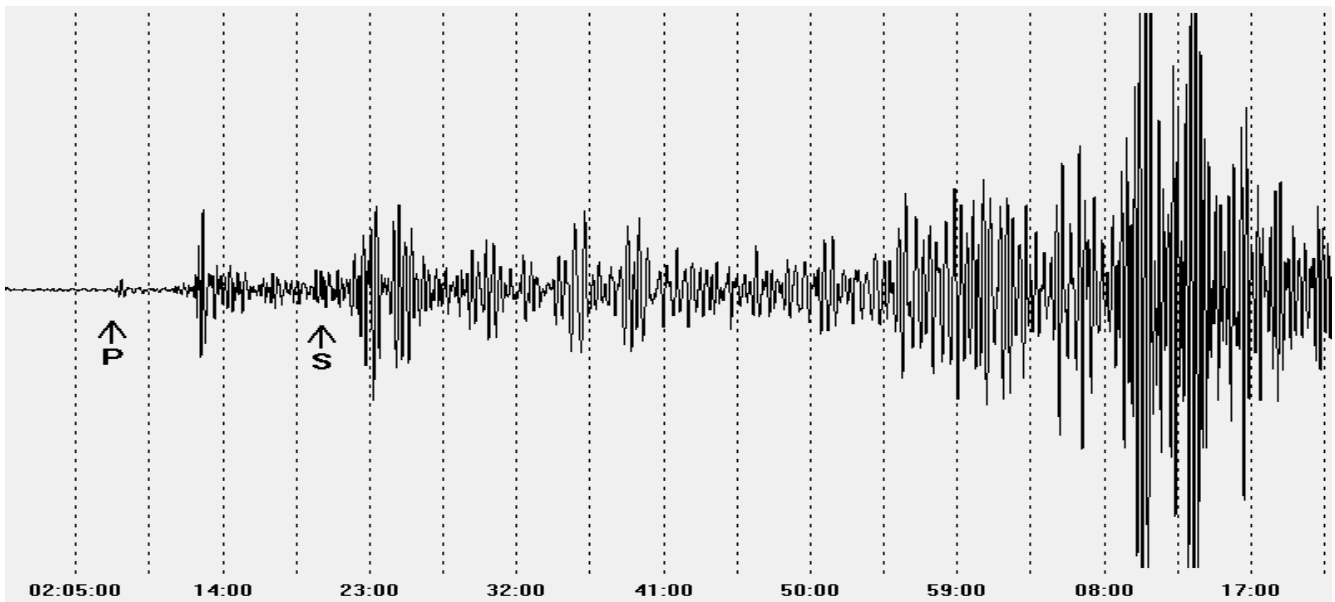
Event coordinates: 42.08° S, 41.83° E, 10 km deep

Delta = 167° (18,609 km)

Passband: 0.04-0.12 Hz.

UTC time indicated across the bottom of the seismogram. Vertical grid spaced at 4.5-minute intervals.

Comments: first visible arrival is PKPdf, at 11:56:05 UTC. Narrower passband used to filter out wind noise.



**FIGURE 19**

**SEISMOGRAM: M6.7 - NIAS REGION, INDONESIA**

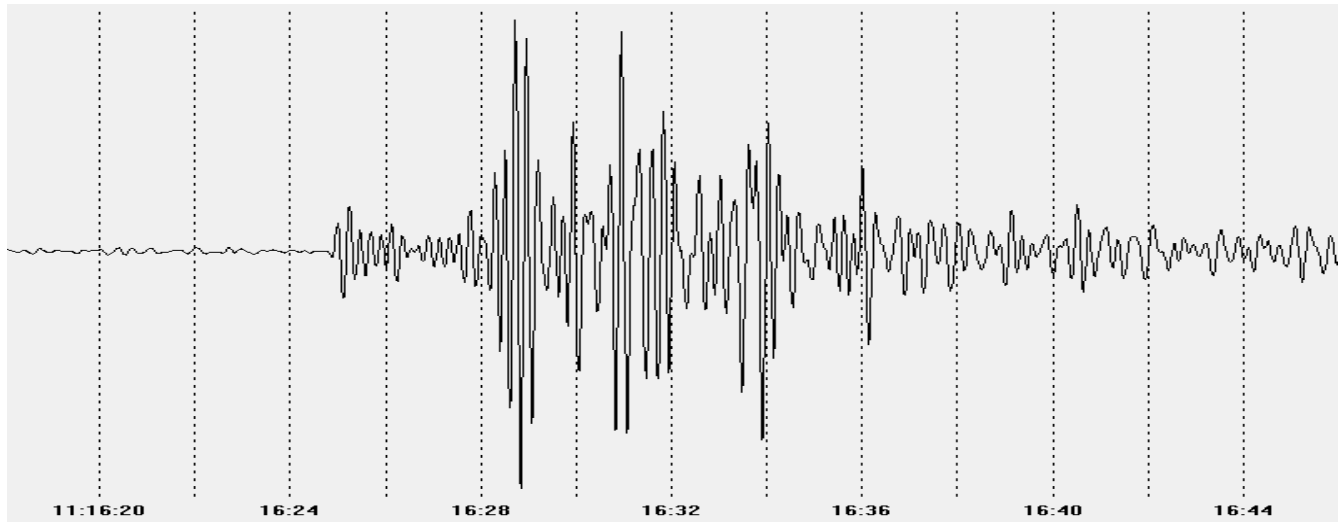
Origin date and time: 2005/07/05, 01:52:04 UTC

Event coordinates: 1.90° N, 97.10° E, 30 km deep

Delta = 120° (13,311 km)

Passband: 0.02-0.10 Hz.

UTC time indicated across the bottom of the seismogram. Vertical grid spaced at 4.5-minute intervals.



**FIGURE 20**

SEISMOGRAM: M2.4 - NEAR WHIDBEY ISLAND, WASHINGTON, USA

Origin date and time: 2005/07/09, 11:16:19 UTC

Event coordinates: 47924° N, 122.396° W, 30.5 km deep

Delta = 0.088° (9.8 km)

Passband: 1-5 Hz.

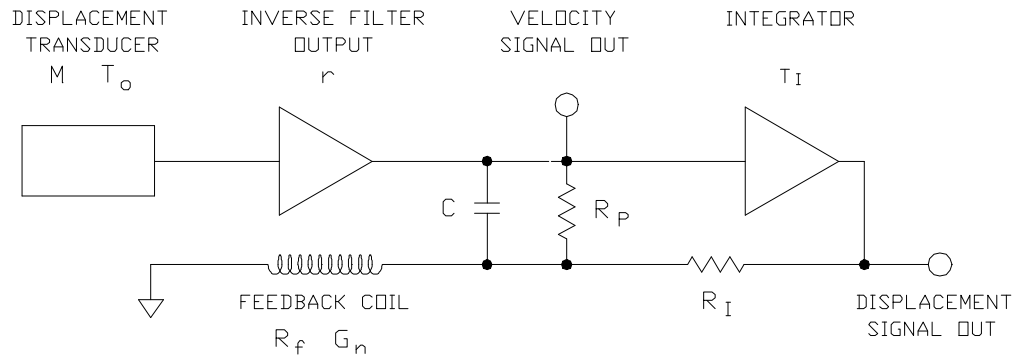
UTC time indicated across the bottom of the seismogram. Vertical grid spaced at 2-second intervals.

Comments: the quasi-integrator's low pass cut-off at ~4Hz, limited the detection of high frequency local events.

# APPENDIX A

## TRANSFER FUNCTION FOR THE MK XX SEISMOMETER.

This Mathcad spread sheet was originally created by Sean-Thomas Morrissey.  
The document was reorganized by Allan Coleman, last revised on 06/18/05



CIRCUIT BLOCK DIAGRAM

$r$  = displacement transducer constant, volts/meter       $r := 129000$

$M$  = sensor mass, kg       $M := .43$

$C$  = feedback capacitor, farads       $C := .000010$

$R_p$  = Proportional (DC) feedback resistor, ohms       $R_p := 800000$

$R_I$  = Integrator feedback resistor, ohms       $R_I := 82000$

$R_F$  = force transducer coil resistance       $R_F := 87$

$T_I$  = Integrator time constant, seconds       $T_I := 100$

$T_o$  = Mechanical free period of sensor, seconds       $T_o := 4.4$

$T_d$  = Displacement Xducer time constant ( $\sim 0$ )

Generator Constant, Newton/Amps       $G \frac{\left| \frac{m}{\text{sec}^2} \right|}{\text{amp}} \quad G_n := 22 \quad G := \frac{G_n}{M} \quad G = 51.1628$

.....function requires m/s/s/amp:=

Damping:=       $S\omega_0 := 2 \cdot \frac{\pi}{T_o} \quad S\omega_0 = 1.4280 \quad \zeta := \frac{1}{2} \cdot \left[ \frac{1}{R_p} + \frac{(S\omega_0)^2}{r \cdot G} \right] \cdot \left( T_I \cdot \frac{R_I}{C} \right)^{.5} \quad \zeta = 0.7059$

Effective "natural Period" (seconds):=       $T_n := 2 \cdot \pi \cdot \sqrt{C \cdot R_I \cdot T_I} \quad T_n = 56.8967$

Generalized Equation       $Y_i := \frac{1}{G \cdot C} \cdot \frac{s_i \cdot (1 + s_i \cdot C \cdot R_F)}{(s_i)^4 \cdot \left( \frac{R_F}{r \cdot G} \right) + (s_i)^3 \cdot \left( \frac{1}{r \cdot G \cdot C} \right) + (s_i)^2 + s_i \cdot \left[ \frac{(S\omega_0)^2}{r \cdot G \cdot C} + \frac{1}{C \cdot R_p} \right] + \frac{1}{C \cdot R_I \cdot T_I}}$

Setting Constants:  $k := \frac{1}{G \cdot C}$   $n := (C \cdot R_F)$   $a := \frac{R_F}{r \cdot G}$   $b := \frac{1}{r \cdot G \cdot C}$   $c := \frac{(S \omega_0)^2}{r \cdot G \cdot C} + \frac{1}{C \cdot R_p}$   $d := \frac{1}{C \cdot R_F \cdot T_I}$

$i := 1, 2, \dots, 600$   $f_i := 10^{(i-300) \cdot 0.01}$   $\omega_i := 2 \cdot \pi \cdot f_i$   $s_i := j \cdot \omega_i$   $s_{200} = 0.6283i$

Evaluating:  $A_i := k \cdot \left[ \frac{s_i \cdot (1 + s_i \cdot n)}{(s_i)^4 \cdot a + (s_i)^3 \cdot b + (s_i)^2 + s_i \cdot c + d} \right]$

$vel_i := \text{Re}(A_i \cdot s_i)$

$\text{maxv} := \text{max}(vel)$

Ref. line, V/m/s:  $\text{maxv} = 1.9574 \times 10^3$

Marking effective periods:  $F_I := \frac{1}{T_I}$   $F_n := \frac{1}{T_n}$

