



Periodic Motion

Physics Lab X

Objective

In this experiment, the variables that affect the periodic motion of two systems, a mass/spring system and a pendulum, will be determined experimentally. Energy Conservation for the mass/spring system will also be checked experimentally.

Equipment List

Table rods with clamps, spring, set of masses, mass hangers, string, set of pendulum bobs, mass scales, pendulum hangers, Dynamics Cart with Track, Two Photogate heads, Smart Timer, Photogate Mount for Dynamics Track.

Theoretical Background

The ability to accurately measure time has produced many technological breakthroughs. In the seventeenth century, the inability to measure time accurately led to major navigational problems for ships crossing the Atlantic Ocean coming to America. In this century, the ability to measure time very accurately, through the use of atomic clocks, has allowed for such technological marvels as the GPS (**G**lobal **P**ositioning **S**ystem) satellite network.

All devices that measure time rely on systems that undergo *periodic motion*. Periodic motion describes the motion of a pendulum (or a mass/spring system) that is displaced from equilibrium. Here equilibrium is the position of the pendulum bob at steady state.

In order for the motion of an oscillating object to be periodic, a *restoring force*, must be present. A restoring force acts to bring an oscillating object back to its equilibrium position. For a pendulum, the restoring force is a portion of the gravitational pull of the earth on the pendulum bob. For a mass/spring system, the restoring force is provided by the spring's resistance to stretch or compress.

For a mass spring system the restoring force is governed by *Hooke's Law*. The mathematical expression,

$$F_{spring} = -k\Delta x \quad (1)$$

shows that the restoring force is proportional to an object's displacement but opposite in sign. In this equation, F_{spring} is the force provided by the spring, Δx is the displacement

of the object connected to the spring (positive if stretched and negative if compressed), and k is known as the *spring constant*, which is a measure of the elastic properties of the spring. The minus sign in this expression is due to the force provided by the spring being in the opposite direction to the stretching or compression. Recall that this force always acts to bring the system to equilibrium.

For a mass/spring system, the period of oscillation (i.e. how long it takes to return from the place it was released) is expressed mathematically as,

$$T_{mass/spring} = 2\pi\sqrt{\frac{m}{k}}. \quad (2)$$

In this expression, $T_{mass/spring}$ is the period of oscillation for the mass/spring system, π is the geometric constant for a circle (i.e. $\pi = 3.1415927\dots$), k is the spring constant described above, and m is the mass at the end of the spring. Note that this period does **not** depend on how far the spring is initially stretched, also known as the amplitude of oscillation. This independence of the period of the oscillation from the amplitude is a key feature in all systems that undergo periodic motion. In part of this lab exercise, this independence on amplitude of the mass/spring system will be tested experimentally. The dependence of the period of the mass spring system on the mass will also be tested experimentally.

Energy conservation for the mass/spring system will also be tested experimentally. Ignoring friction, for energy to be conserved, the initial and final energies of the system must be equal,

$$E_i = E_f. \quad (3)$$

Energy is the sum of the kinetic and potential energies. Applying this to the previous equation,

$$E_i = E_f, E = K + P \implies K_i + P_i = K_f + P_f. \quad (4)$$

There is no rotational component for the mass/spring system, therefore the only type of motion is translational motion. For this experiment, the mass/spring system will be arranged horizontally so that the only source of potential energy is due to the compression of the spring. Applying this to the previous equation,

$$K = \frac{1}{2}mv^2, P = \frac{1}{2}kx^2 \implies \frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2. \quad (5)$$

If this mass is released from rest a distance A away from the equilibrium position, the previous equation reduces to,

$$\frac{1}{2}kA^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2. \quad (6)$$

The distance A is the oscillatory amplitude of the mass in the mass/spring system. With this equation, the speed of the mass, v_f , can be determined for any distance x_f , away from the equilibrium position. Solving this equation for the speed yields,

$$v_{theo} = \sqrt{\frac{k}{m}(A^2 - x_f^2)}. \quad (7)$$

This result will be tested experimentally in this series of experiments.

For a pendulum the restoring force is governed by the gravitational force, and the following expression describes the period of oscillation of a pendulum:

$$T_{\text{pendulum}} = 2\pi\sqrt{\frac{L}{g}}. \quad (8)$$

In this equation, T_{pendulum} is the period of the pendulum, L is the length of the pendulum, and g is the constant acceleration due to gravity ($g = 9.80 \text{ m/s}^2$). The period here is also independent of how far the bob is displaced, but in this case it is only accurate for small displacements (i.e. less than $\sim 15^\circ$). Note that this period is also independent of the mass of the bob. This independence will also be tested experimentally. The dependence of the period on the length of the pendulum will be tested experimentally.

Procedure

Be sure to convert all units to kilograms, meters, and seconds before performing calculations.

Mass/Spring System

Determination of the Spring Constant

In this section the spring constant, k , will be determined experimentally.

1. Turn on the computer at your workstation if it is not already on. Open the file titled **Spring Constant Calc.** from your desktop. If you do not see an icon for **Spring Constant Calc.** your lab instructor will need to load the program onto your computer.
2. Place one of the springs at your station on the crossbar hook. Place a mass hanger on the free end of the spring.
3. Place 50 g on the hanger. Measure the distance from the crossbar hook to the bottom of the mass hanger and enter this measurement as y_0 on the spreadsheet.
4. Place an additional 20 g on the mass hanger. Measure the distance from the crossbar hook to the bottom of the mass hanger and enter this measurement as y_1 on the spreadsheet.
5. Repeat step 4 six times (add 20 g to the hanger, measure the distance from crossbar hook to mass hanger and record distance).
6. Repeat steps 2-5 for the remaining spring.
7. Print the spreadsheet; include it with your lab report.

Dependence of Period on Amplitude

In this section the amplitude of an oscillating mass/spring system will be varied.

1. Determine the mass of the cart and record the value from the scale as M_{Cart} on the data table.
2. Attach the springs to both ends of the cart. Attach the free end of the springs to the track's bumper-posts. If necessary consult with your lab instructor for further instructions.
3. Place the photogate flag in the cart. Adjust the height of the photogate so that the light sensor is approximately centered with the center of the photogate flag.
4. Plug the photogate into a smart timer.
5. Set the Smart Timer to the **Time:Pendulum** mode. Pull the cart 5 *cm* from the equilibrium position. Record this distance as the amplitude of oscillation A , on your data sheet. Press the black key on the timer. Release the cart.
6. After the cart has passed through the gate twice, a time should appear on the timer. This displayed value is half the period of oscillation of the mass. Record this time as $\frac{1}{2}T_{exp}$ on your data sheet.
7. Repeat steps 5-7 four times for amplitudes of 10 *cm*, 15 *cm*, 20 *cm*, and 25 *cm*.

Dependence of Period on Mass

In this section the mass of an oscillating mass/spring system will be varied.

1. Record the mass of the cart. Set the Smart Timer to the **Time:Pendulum** mode. Pull the cart 20 *cm* from equilibrium and record the half-period.
2. Place a 100 *g* mass into the cart. Set the timer to record the half-period. Pull the cart 20 *cm* from equilibrium. Release the cart and record the half-period on your data table.
3. Repeat step 2 for masses quantities 200 *g*, 300 *g*, 400 *g* and 500 *g*.

Energy Conservation

In this experiment, you will take a series of measurements to verify energy conservation for the mass/spring system.

1. Remove all masses from the cart's flat-bed. Record the cart's mass m and the total spring constant k_T in the provided spaces. Set the timer to the **Speed:One Gate** mode. Record the position of the photogate as 0.0 *cm* in the x_f column of your data sheet.

2. Pull the cart 20 *cm* from the equilibrium position. Press the black button on the timer and prepare to measure and record the speed. Release the cart. The speed of the cart through the equilibrium point x_f is displayed on the Smart Timer. Record this speed on your data sheet as v_{exp} . If the speed is not displayed on the timer, consult with the lab instructor.
3. Move the photogate 4 *cm* from the equilibrium position. Record this distance in the x_f column of your data sheet.
4. Pull the cart 20 *cm* from the equilibrium position. Press the black button on the timer. Release the cart and record the speed displayed on the timer on your data sheet.
5. Repeat steps 3 and 4 for x_f values of 8 *cm*, 12 *cm*, and 16 *cm*.

Pendulum

Dependence of Period on Bob Mass

In this section the mass of an pendulum bob will be varied.

1. Unplug the photogate from the Smart Timer. Plug the photogate attached to the balsa wood rod to the Smart Timer.
2. Select one of the pendulum bobs from the set on the lab table. Use the mass scale to measure the mass of the bob; record the value on your data sheet.
3. Detach the string from the cross bar and feed the string through the bob (You may wish to tie the string to a washer). Re-attach the string to the cross bar.
4. Adjust the length of the string to 70 *cm*. The string length should be measured from the cross bar hook to the center of the pendulum bob. Consult with the lab instructor if you need assistance.
5. Adjust the position of the photogate so that light sensor is approximately centered with the center of the pendulum bob. Set the timer to the **Time: Pendulum** mode.
6. Displace the pendulum bob 5 to 10 *cm* from equilibrium. Press the black button on the timer. Release the bob. The period of the pendulum should be displayed on the timer. Record this value as T on your data sheet.
7. Detach the string from the cross bar and cut the pendulum bob away from the string. Select a different pendulum bob from the set at your station. Measure its mass and attach it to the string.
8. Repeat steps 2-7 for a total of four different pendulum bobs.

Dependence of Period on the Pendulum Length

In this section the length of a unstretchable, massless string attached to a pendulum bob will be varied.

1. Copy the last length and period data from the previous section into the first row of your data table for this section.
2. Adjust the length of the string to 60 *cm* and record the measured length on your data table.
item Displace the pendulum bob 5 to 10 *cm* from equilibrium. Set the smart timer to the **Time:Pendulum** mode.
3. Repeat step 2 and 3 for pendulum lengths of 50 *cm*, 40 *cm* and 30 *cm*.

Data Analysis

Mass/Spring System

Determination of Spring Constant

Note: Be sure to print the Spring Constant spreadsheet and include this with your lab report.

Dependence of Period on Amplitude

1. For each value of the amplitude A , calculate and record the oscillation period T_{exp} by multiplying $\frac{1}{2}T_{exp}$ by two.
2. Calculate the percent variation in the oscillation periods using

$$\% \text{ variation} = 100 \times \frac{\text{maximum value} - \text{minimum value}}{2 \times \text{average value}}. \quad (9)$$

Dependence of Period on Mass

1. Calculate the total spring constant for the mass/spring system using the average spring constant for each of the springs and,

$$k_T = k_{1,ave} + k_{2,ave}. \quad (10)$$

2. For each value of the mass m , calculate and record the period of oscillation T_{exp} by multiplying $\frac{1}{2}T_{exp}$ by two.
3. For each value of the mass m , calculate the theoretical value of the period T_{theo} using the total spring constant, k_T and,

$$T_{theo} = 2\pi \sqrt{\frac{m_c + m}{k_T}}. \quad (11)$$

- Calculate the percent difference between the average experimental value of the period, for each mass value, and the theoretical value using,

$$\% \text{ difference} = 100 \times \frac{\text{theoretical value} - \text{experimental value}}{\text{theoretical value}}. \quad (12)$$

- Calculate the percent variation in the average experimental values of the period, T_{exp} .
- Graph the square of the experimental period, T_{exp}^2 , as a function of the mass, m . Draw the straight line that comes closest to your data points and determine the slope and y-intercept of this line.

Conservation of Energy

- Using the initial amplitude of the cart, A , calculate the total energy of the mass spring system, which is also the initial potential energy of the system since it was released from rest, using,

$$E_T = P_i = \frac{1}{2}k_T A^2. \quad (13)$$

- For each value of displacement from equilibrium, x_f , calculate the theoretical value of the speed of the cart using,

$$v_{theo} = \sqrt{\frac{k_T}{m}(A^2 - x_f^2)}. \quad (14)$$

- Calculate the percent difference between the experimental and theoretical values of the speed of the cart.
- Calculate the final kinetic energy of the cart for each value of x_f using,

$$K_f = \frac{1}{2}mv_{exp}^2. \quad (15)$$

- Calculate the final potential energy of the cart for each value of x_f using,

$$P_f = \frac{1}{2}k_T x_f^2. \quad (16)$$

- Calculate the total energy of the system for each value of x_f using,

$$E_T = K_f + P_f. \quad (17)$$

- For each value of x_f , calculate the percent difference between the theoretical value of the total energy, $E_T = \frac{1}{2}k_T A^2$, and the experimental value of the total energy, $E_{Total} = K_f + P_f$.
- Calculate and record the percent variation in the total energy, $E_T = K_f + P_f$.

- Estimate the uncertainty in each measured quantity, x , A , T , and v . Use these values, and the smallest measured value of each quantity, to calculate the percent uncertainty in each.

$$\% \text{ Uncertainty} = 100 \times \frac{\text{Uncertainty}}{\text{Smallest Measured Value}} \quad (18)$$

- Record the largest percent variation in the spring constant k_1 and k_2 and the percent uncertainty in the spring constant.
- Record the largest percent uncertainty in the experiment in the space provided.

Pendulum

Dependence of the Period on the Bob Mass

- Calculate the percent variation in the oscillation periods.

Dependence of Period on the Pendulum Length

- For each value of the pendulum length L , calculate the theoretical value of the pendulum period using $g = 9.80 \text{ m/s}^2$ and,

$$T_{\text{theo}} = 2\pi \sqrt{\frac{L}{g}}. \quad (19)$$

- For each value of the pendulum length, calculate the percent difference between the experimental period and the theoretical period of the pendulum.
- Calculate the percent variation in the experimental values of the pendulum period.
- Estimate the uncertainty in the measured quantities m , T , and L . Use these values, and the smallest value of each, to calculate the percent uncertainty in each quantity.

$$\% \text{ Uncertainty} = 100 \times \frac{\text{Uncertainty}}{\text{Smallest Measured Value}} \quad (20)$$

- Record the largest percent uncertainty in m , T , and L as the largest percent uncertainty in the experiments.
- Graph the square of the experimental period, T_{exp}^2 as a function of the length of the pendulum, L . Draw the straight line that comes closest to these points and determine the slope and y-intercept of this line.

Questions

1. Based on your experimental measurements, does the period of the mass/spring system depend upon the amplitude of the oscillations? Why or why not? Should it according to the theory?
2. Based on your experimental measurements, does the period of the pendulum depend on the mass of the pendulum bob? Why or why not? Should it according to the theory?
3. From your data, was energy conserved in the mass/spring system? Why or why not?
4. What effect would friction have on the measured speed of the cart when compared with the theoretical value of the speed? Is this effect consistent with your data?
5. In the following figure, a 10 kg weight is suspended from the ceiling by a spring. The weight-spring system is at equilibrium with the bottom of the weight about 1 m above the floor. The spring is then stretched until the weight is just above the eggs. When the spring is released, the weight is pulled up by the contracting spring and then falls back down under the influence of gravity. On the way down, it



- (a) reverses its direction of travel well above the eggs.
- (b) reverses its direction of travel close to the top of the eggs.
- (c) make a mess as it crashes into the eggs.

[Taken from Mazur, **Peer Instruction: A User's Manual**, p. 137]

6. Does a car bounce on its spring faster when it is empty or when it is fully loaded? Explain your reasoning. [Taken from Giancoli, **Physics**, 5th ed. p. 341]
7. If a pendulum clock is accurate at sea level, will it gain time, lose time, or remain accurate when taken to high altitude? Explain your reasoning. [Taken from Giancoli, **Physics**, 5th ed. p. 341]
8. A tire swing hangs from a branch nearly to the ground. How could you estimate the height of the branch using only a stopwatch? [Taken from Giancoli, **Physics**, 5th ed. p. 341]