



Conservation of Linear Momentum

Objective

In this series of experiments, the conservation of linear momentum and kinetic energy will be tested for different types of collisions.

Equipment List

Air track, two air track carts with flags and magnetic inserts, set of masses, two photogates and smart timers, computer, mass scales.

Theoretical Background

In the previous lab exercise, conservation of energy was explored. In this lab exercise, another conservation principle, the conservation of momentum, will be explored.

Momentum \vec{p} is defined as the product of the mass of an object m and its velocity \vec{v} :

$$\vec{p} = m\vec{v} \quad (1)$$

Note that, since velocity is a vector, momentum is also a vector. Conservation of momentum is most useful when considering colliding objects. Momentum being conserved means that the amount of momentum a set of objects has before a collision is the same after the collision. This can be expressed mathematically as

$$\vec{p}_i = \vec{p}_f \quad (2)$$

where \vec{p}_i is the initial momentum and \vec{p}_f is the final momentum. For momentum to be conserved, no net external force must act on the objects. However, any force that acts between the objects (i.e. internal forces) will not affect momentum conservation, since, by Newton's third law, this force must affect both equally in magnitude but in opposite directions. The gravitational attraction between the

masses, for example, would not affect momentum conservation. If, however, the masses were on an inclined plane, the net external force pulling both masses down the plane would cause momentum not to be conserved.

If no net external force acts on the two colliding objects, so that momentum is conserved, then there are two cases for how the objects can collide. In the first case, the objects can collide and bounce off each other while conserving kinetic energy. This type of collision is known as an elastic collision. For this type of collision, combining Equations 1 and 2, the conservation of momentum can be written as,

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \quad (3)$$

where m_1 is the mass of the first object, m_2 is the mass of the second object, \vec{v}_{1i} is the initial velocity of the first mass, \vec{v}_{2i} is the initial velocity of the second mass, \vec{v}_{1f} is the final velocity of the first mass, and \vec{v}_{2f} is the final velocity of the second mass. Since kinetic energy is also conserved,

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad (4)$$

Solving equations (3) and (4) together yields the velocities of m_1 and m_2 after the collision.

$$\vec{v}_{1f} = \frac{m_1 - m_2}{m_1 + m_2} \vec{v}_{1i} + \frac{2m_2}{m_1 + m_2} \vec{v}_{2i} \quad (5a)$$

$$\vec{v}_{2f} = \frac{2m_1}{m_1 + m_2} \vec{v}_{1i} + \frac{m_2 - m_1}{m_1 + m_2} \vec{v}_{2i} \quad (5b)$$

In the other case, the objects can collide and stick together. This type of collision is known as an inelastic collision. In an inelastic collision, the kinetic energy is not conserved. Instead, some of the initial energy goes into other forms, such as heating the objects. For inelastic collisions, conservation of momentum can be written as,

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2) \vec{v}_f \quad (6)$$

where \vec{v}_f is the final velocity of the two masses.

For this lab exercise, elastic and inelastic collisions between two carts on an air track will be explored to see if momentum, and kinetic energy, is conserved.

Procedure

In all cases, one of the air track cart (cart 2) is initially at rest and the other one (cart 1) approaches it with a moderate velocity set by hand.

Initial Check

1. Make sure that the flags on each of the carts are orientated properly.
2. The photogate sensors should be positioned about 40.0 cm apart from each other and at a proper height for the flags.
3. The photogate sensors should be plugged into two smart timers respectively. Set both smart timers in “Velocity” measurement mode with “One Gate”.
4. The air track should be leveled.

Inelastic Collisions

1. Use the mass scale to measure the masses of the air track carts, including the flags and the magnetic inserts. Record the measured m_1 and m_2 on the data sheet.
2. Make sure that the magnetic inserts on each cart are attracted to each other.
3. Place m_2 in the middle of the two photogate sensors. Keep it stationary.
4. When you are ready, press the black keys on the smart timers so that an asterisk (*) is shown on each timer’s LCD screen. Then set m_1 in motion with a moderate speed.
5. Record the speed of m_1 (v_{li}) when it passes the first photogate and the speed of m_1+m_2 (v_f) when they pass the second one.
6. Repeat step 1~5 five times with different v_{li} (by pushing differently) and record the results on the datasheet.
7. Repeat step 1~6 with different mass arrangement as shown in Table 1.

Table 1

| case | m1 | m2 |
|------|-------------|-------------|
| 1 | cart | cart |
| 2 | cart | cart + 200g |
| 3 | cart + 200g | cart |

Elastic Collisions

1. Flip the magnetic insert on one of the carts, so that the inserts are repelling each other instead of attracting.

2. Repeat Step 1~6 in the previous section for three different cases as listed in Table 1.

Note: Students need to think about how to measure v_{1f} in elastic collisions.

Data Analysis

Inelastic Collisions

1. Calculate the percent difference between the theoretical and experimental values of v_f using the following equation for theoretical values.

$$v_f = \frac{m_1}{m_1 + m_2} v_{1i}$$

2. Calculate the initial and final total linear momentum (P_i and P_f), as well as the kinetic energies (KE_i and KE_f).
3. Plot P_f as a function of P_i for each case. Determine the slope of this line and record it on your data sheet.

Elastic Collisions

1. Calculate the percent difference between the theoretical and experimental values of v_{1f} and v_{2f} . Use Equation (5) to determine the theoretical values.
2. Calculate the initial and final total linear momentum (P_i and P_f), as well as the kinetic energies (KE_i and KE_f).
3. Plot P_f as a function of P_i for each case. Determine the slope of this line and record it on your data sheet.
4. Plot KE_f as a function of KE_i for each case. Determine the slope of this line and record it on your data sheet.

Selected Questions

1. Suppose the magnetic insert on the carts was replaced with velcro to hold the carts together when they collided. What effect would the velcro have on the conservation of momentum between the two carts? Explain your reasoning.
2. Suppose the air track was tilted. What effect would this tilt have on conservation of momentum between the two carts? Explain your reasoning.
3. How does friction affect the conservation of momentum in this experiment? Explain your reasoning.
4. Suppose magnets were set on two air track carts, $m_{\text{left}}=100\text{g}$ and $m_{\text{right}}=150\text{g}$, so that they repelled instead of attracted and that the two carts were initially tied together. The two carts were originally moving at a speed of 10.0cm/s to the left, and the final speed of the cart on the left (m_{left}) is 30.0 cm/s after the string holding the two carts together snaps. What is the speed of the cart on the right (m_{right}) after the string snaps? (Hint: Consider the formula for elastic collisions.)
5. Show that, if: (1) the second cart has no initial velocity, (2) the carts collide inelastically, and (3) the carts were of equal mass, then the ratio of the final kinetic energy to the initial kinetic energy is $1/2$.