



# Hydrostatics

## Physics Lab XI

### Objective

Students will discover the basic principles of buoyancy in a fluid. Students will also quantitatively demonstrate the variance of pressure with immersion depth in a fluid.

### Equipment List

#### Buoyancy

Several cylindrical bottles of various volumes and masses, a collection of pennies (all pennies should be of one time period), a water source, two 25 mL plastic graduated cylinders, Aluminium and Copper cylinders, a mass scale, one pair of digital calipers.

#### Variation of Pressure with Immersion Depth

The WL 1080-90 Boyle's Law Apparatus consisting of two Plexiglas tubes, one containing water and a smaller graduated tube used to measure the change in length of an air column existing within the small tube.

### Theoretical Background

#### Buoyancy

In this set of exercises, you will discover the basic principles governing buoyancy in a fluid. The theoretical relation that is needed for these experiments is the relationship of an material's mass and its volume. The density of the material is stated by the equation

$$\rho = M/V \quad (1)$$

where  $M$  is the mass of the material,  $V$  is its volume, and  $\rho$  is its density.

## Variation of Pressure with Depth

For this exercise you will verify the notion that the pressure exerted on an object varies with regards to the object's immersion depth in a fluid. The material submerged in the fluid experiences a force due to the weight of the fluid, which in this exercise is water, directly above the object as shown in figure 1 where the object shown is a submarine.

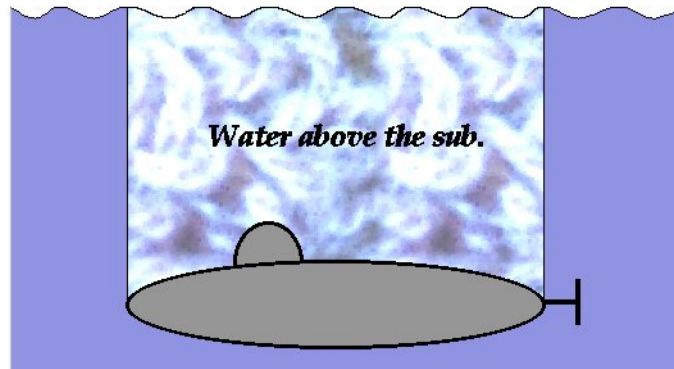


Figure 1: Section of water above the sub

The force exerted on the submarine is equal to the mass of the water above it,

$$f_{water} = m_{water}g. \quad (2)$$

When working with fluids, it is more convenient to work with the density rather than the mass. Using the density relationship,

$$f_{water} = m_{water}g = \rho_{water}V_{water}g. \quad (3)$$

The volume of water above the submarine  $V_{water}$  can be written in terms of the depth of the submarine  $h$  and the cross-sectional area of the sub  $A$

$$f_{water} = \rho_{water}hAg. \quad (4)$$

Dividing both side of equation 4 by the cross-sectional area of the sub, and using the definition of pressure ( $P = \frac{F}{A}$ ), we obtain

$$\frac{f_{water}}{A} = P = \rho gh. \quad (5)$$

In this exercise, the relationship given by equation 5 will be tested.

## Procedure

### Buoyancy

#### Experiment # 1

1. Fill both plastic graduated cylinders with 15 *mL* of water.
2. You should have two small metal cylinders at your lab station. One of the cylinders should be noticeably less massive than the other, but both cylinders should be the same size (i.e. both should have the same volume). On the data sheet under the heading *Experiment # 1*, write down your prediction of what will happen when the cylinders are placed in the graduated cylinders with water. Write this in the box titled *Experimental Prediction*. In particular, which metal cylinder (the lighter one or the heavier one) will cause the water level to rise the most when dropped into the tubes? In the box titled *Reasoning* write down the reasoning behind your prediction.
3. Carefully place the metal cylinders into the tubes of water, one cylinder in each tube. Note the rise in the water level of both tubes. Record this experimental result in the box title *Experimental Result*.
4. Based on the results of the experiment, reevaluate your prediction and reasoning. In particular, based on the experimental results, was the displaced water responding to the mass or the volume of the cylinders? Write down the answer this question, along with the reevaluation of your prediction and reasoning in the box titled *Conclusions*.

#### Experiment # 2

1. Take the glass beaker at your lab station and fill it approximately 2/3 full of tap water.
2. Using the mass scale, sort the pennies on the lab table so that you have about 50 pennies of the same mass.
3. Using the caliper, measure the length and diameter of one of the small plastic cylindrical bottles at your lab station. For the diameter, measure the bottle's diameter, excluding the diameter of the cap. Using these measurements, calculate and record the volume of the bottle. Note that the volume of a cylinder is given by:

$$V_{cylinder} = \pi \frac{d^2}{4} \ell \quad (6)$$

4. Fill the bottle approximately 2/3 full of pennies. Measure and record the mass of the bottle filled with pennies.
5. Calculate and record the density of the bottle 2/3 full of pennies. Density is defined as

$$\rho = \frac{M}{V} \quad (7)$$

6. Using the volume of the bottle, and the density of water ( $\rho_{water} = 1.0 \times 10^3 \text{ kg/m}^3 = 1.0 \text{ g/cm}^3$ ), calculate and record the mass of the water displaced by the bottle if it were completely submerged.
7. Place the bottle in the beaker of water. Note whether or not the bottle sinks or floats on your data table.
8. Write down in the spaces provided a comparison between the mass of the bottle and the maximum mass displaced by the water, and compare the density of the object with the density of water.
9. Repeat this process for the same bottle filled 1/4 full of pennies.
10. Based on these experiments, write down in the space provided your observations for the requirements of floatation. In particular if you could know only one thing about a group of objects of different masses and volumes, what quantity would you most desire in order to distinguish between floating object and sinking objects?

### Experiment #3

1. Measure and record the mass of a single penny.
2. Measure and record the mass of the empty bottle.
3. Based on the mass of a single penny and the mass of the empty bottle used in the previous experiment, determine the maximum number of pennies that can be placed in the bottle so that the bottle floats. If your calculation contains a fraction, round the value down to the nearest whole integer value, for example if your calculated value for maximum number of pennies is 12.8, record 12 pennies max. are required to float the bottle.
4. Place the calculated number of pennies in a bottle. Place the bottle in the beaker to determine if it will float with this quantity of pennies. If the bottle does not float, review your calculations and discuss possible errors with your lab instructor or TA.
5. After determining the maximum number of pennies for which the bottle remains buoyant in water, put one more penny in the bottle. Now place the bottle in the beaker and observe the bottle's reaction. If the bottle floats, recheck your calculations and discuss possible errors with your lab instructor or TA.

### Experiment # 4

Have each member of your lab group repeat Experiment #3 with a different bottle. Each member's bottle should have a different mass and volume, therefore it will be necessary to determine the mass displacement of water due to the new bottle density. If necessary, ask other lab groups if you can borrow their unused bottles.

## Variation of Pressure with Depth

1. Turn on the computer at your workstation if it is not already on. Open the file titled **Hydrostatics Lab** from your desktop. If you do not see an icon for **Hydrostatics Lab** your lab instructor will need to load the program onto your computer.
2. Enter the current atmospheric pressure  $P_{atm}$  in units of inches of mercury on the spreadsheet. This value will be provided by the lab instructor.
3. Enter the total length of the inner tube  $L_o$  on the spreadsheet. (It may be helpful to invert the tube so that the closed end faces the ceiling.) The inner tube has millimeter graduations. **Be sure to convert your measurement to meters!!!**
4. Pour 1200 mL of tap water into the outer tube. Immerse the open end of the inner tube in the outer tube. The initial depth  $d$  of the inner tube is 100 mm. Use the graduations of inner tube for this measurement. Enter the measured immersion depth value in the depth  $d$  column of the spreadsheet.

**NOTE: Be careful not to let any air escape from the inner tube once it has been immersed! Notify the lab instructor if you suspect a air/water leakage of your apparatus.**

5. Enter the length of the water column, as  $L_w$  on the spreadsheet. Figure 2 illustrates the variables  $L_o$ ,  $d$  and  $L_w$  in relationship to the tube.

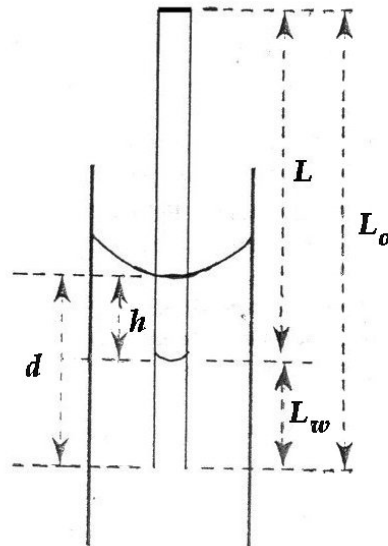


Figure 2: Immersion of inner tube in outer tube.

6. Increase the immersion depth by intervals of 50 mm. Enter the immersion depth  $d$  and the corresponding water column length  $L_w$  for each depth interval on the spreadsheet. The maximum immersion depth is 75 mm. Stop collecting data once you've reached a depth of 75 mm.

## Data Analysis

### Variation of Pressure with Depth

1. The ideal gas law may be applied as the equation of state of the container of gas. Assuming the container is insulated and no work is done on the system, the following equation may be a proper statement:

$$PV = NkT. \quad (8)$$

Here the product of pressure and volume is equal to the product of the number of gas molecules  $N$ , the absolute temperature of the gas  $T$  and Boltzmann's constant  $k$ . Note, the pressure  $P$  is that of the gas and the volume  $V$  is that of the tube. If the number of molecules of gas in the tube remains constant (i.e. the tube does not leak air), and the temperature remains fixed, then the product of pressure and volume also remains constant. Therefore the pressure on the gas in the tube when the tube is submerged should be equal to the product of the pressure of the unsubmerged tube's end (i.e. atmospheric pressure) and the original volume of the tube divided by the submerged volume of the tube

$$\begin{aligned} P_{atm}V_o = NkT = constant \Rightarrow P_{sub} &= \frac{P_{atm}V_o}{V_{sub}} = \frac{P_{atm}\pi L_o \frac{D^2}{4}}{\pi L_{sub} \frac{D^2}{4}} \\ &\Rightarrow P_{sub} = \frac{P_{atm}L_o}{L}. \end{aligned} \quad (9)$$

Here the pressure exerted on the submerged gas  $P_{sub}$  is the experimental pressure  $P_{exp}$ .

2. The **Hydrostatics Lab** program is designed to calculate the experimental and theoretical pressure values. Inform your lab instructor if the program does not produce valid solutions for your data.
3. After recording all measurements for  $d$  and  $L_w$ , select the **P vs h graph** tab. The program produces a plot of the data you recorded.
4. **Print** one each of the spreadsheet and the graph plot. Include these prints with your lab report.

## Selected Questions

1. A cube with a mass of  $86\text{ g}$  and a volume of  $40\text{ cm}^3$  is placed in a variety of liquids described below. In which, if any, of these liquids will the cube float?
  - (a) A liquid with a mass of  $95\text{ g}$  and a volume of  $80\text{ cm}^3$
  - (b) A liquid with a mass of  $140\text{ g}$  and a volume of  $200\text{ cm}^3$
  - (c) A liquid with a mass of  $60\text{ g}$  and a volume of  $50\text{ cm}^3$
  - (d) A liquid with a mass of  $50\text{ g}$  and a volume of  $20\text{ cm}^3$
  - (e) The cube will not float in any of these liquids.

Circle your answer choice.

2. An odd shaped solid sinks in a liquid whose density is  $4.0\text{ g/cm}^3$ . If the mass of the object is  $32\text{ g}$  and its volume is  $6\text{ cm}^3$ , determine the mass of liquid displaced by the solid.
3. Suppose the tube had a small leak in it that allowed some of the air in the gas bubble to escape as it was submerged. What effect would this have on the experimental value of the pressure?  
Explain your reasoning.
4. Suppose the tube had been submerged in hot water (i.e. well above room temperature), so that the gas bubble was heated as it was submerged. What effect would this have on the experimental value of the pressure?  
Explain your reasoning.