Analysis of Experimental Uncertainties: Density Measurement
Physics Lab II

Objective

This laboratory exercise allows students to estimate and analyze experimental uncertainties. Students will calculate the density of brass and chrome. Conclusions are drawn from correlations between accepted density values and experimental values.

Equipment List

Electronic Digital Caliper, ACCULAB VI-200 mass scale, a set of masses of different shapes and composition; 5 chrome spheres, 4 brass cylinders

Theoretical Background

In this set of experiments, you will determine the density of various objects. Density is related to the mass and volume by

$$\rho = M/V \tag{1}$$

where M is the mass of the object, V is the volume of the object, and ρ is the density of the material.

For this set of experiments, the measured objects are spheres, and cylindrical disks, the latter consisting of single holes drilled out of their centers. The volume of the spheres is determined by:

$$V_{sphere} = (4/3)\pi R^3 = (4/3)\pi (D/2)^3 = (1/6)\pi D^3. \tag{2}$$

In this set of equations, V_{sphere} is the volume of the sphere, R is the radius of the sphere, and D is the diameter of the sphere, which is twice the radius R.

For the cylindrical masses, the volume can first be approximated by the volume of a cylinder,

$$V_{cylinder,1} = \pi R^2 h = \pi (D/2)^2 h = (1/4)\pi D^2 h. \tag{3}$$

In this equation, $V_{cylinder,1}$ is the first approximation to the volume of the cylinder, R is the radius of the cylinder, D is the diameter of the cylinder, which is twice the radius of the cylinder, and h is the height of the cylinder.

The more exact formula for the volume of the cylinder subtracts out the volume of the hole,

$$V_{cylinder,2} = \pi(R^2 - R_{hole}{}^2)h = \pi((D/2)^2 - (D_{hole}/2)^2)h = (1/4)\pi(D^2 - D_{hole}{}^2)h. \quad (4)$$

In this equation, $V_{cylinder,2}$ is the more exact representation for the volume of the cylinder, h is the height of the cylinder, R is the outer radius of the cylinder, R_{hole} is the radius of the hole, D is the outer diameter of the cylinder, which is twice the outer radius of the cylinder, and D_{hole} is the diameter of the hole, which is twice the radius of the hole.

The measurement of the density of various objects provides the opportunity to introduce methods used to analyze scientific data. Learning to use these methods is the primary purpose of this experiments. When taking measurements of a physical quantity, various effects may cause the experiments to deviate from a more idealized case set out in theory. These types of effects introduce *experimental uncertainty*, or experimental error, into the experiment **The errors referred to here are not "mistakes" in the more common usage of the word, but departures from the idealized world of theory. For this reason, the term "Human Error" should never be used in a scientific report.**

There are two classes of experimental uncertainties: *systematic errors* and *random errors*. Systematic errors cause the data to be shifted in the same direction away from the theoretical ideal. Examples of effects that cause systematic errors include friction slowing down an object, or an uneven table top causing an object to speed up. Random errors cause individual measurements to vary around some average value. Examples of effects that cause random errors include not releasing an object from exactly the same release point, or looking at the scale on an instrument at a slightly different angle each time a reading is recorded as measured data. Being able to distinguish between these two sources of experimental error is one of the goals of this lab exercise.

For the experiments in this class, there are two basic methods that will be used to compare theory with experiment. The first basic method is a comparison of an experimentally measured quantity to a previously determined, or theoretically calculated, quantity. This simple method is a *percent difference calculation*, and is defined by,

$$\% \ difference = 100 \times \left(\frac{given - measured}{given}\right). \quad (5)$$

Another method of uncertainty analysis looks at the variation of a quantity. First an average of a group of data is taken. The difference between the largest value and the smallest is calculated, and this difference is then divided by twice the average value and multiplied by 100 to yield the *percent variation* in the data set:

$$\% \ variation = 100 \times \left(\frac{largest \ value \ - \ smallest \ value}{2.0 \times average}\right). \quad (6)$$

The percent variation is useful in looking at how much an experimental quantity varies. If a theory suggests that a particular quantity is constant, then that quantity should have a small percent variation.

Another way of determining if a theory is supported by experimental data is through the use of *graphs*. In a typical graph, data from experiments is plotted as individual points, then a line is drawn that comes closest to these points This line may be either curved or straight, though a straight line is the most common. If the characteristics of this curve match the characteristics of the theoretically obtained formula, then the experiment supports the theory. If a straight line is plotted, the slope and y-intercept of this line are compared to the slope and y-intercept expected from the theoretical relationship.

Procedure

Remember to include all units! Convert all measurements to meters, kilograms, and seconds!!!

Density of Spheres

1. Power-up the computer at your station if it is not already on. Open the file titled **Density Lab** on your desktop.

2. Select the tab entitled **Data Sheet for Spheres**. All measurements performed using spheres will be recorded on this spreadsheet.

3. Select one of the spheres. Measure its mass using the scale and record this in the **mass** column.
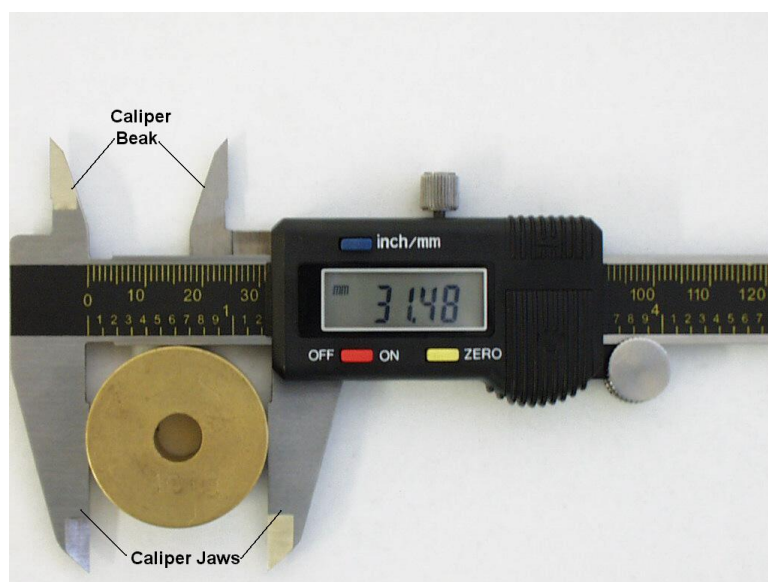

Figure 1: Electronic Digital Caliper

Using the Electronic Digital Caliper

The following subtext provides instructions for proper use of the digital caliper.

(a) Turn on the Electronic Digital Caliper. Be sure to set the caliper to measure metric units.

(b) It may be necessary to zero adjust the caliper. If your caliper does not display "0.00 mm" at start up, press the "zero setting" button.

(c) Figure 1 displays the measured diameter of a cylinder placed between the jaws of the caliper. Notice the cylinder is positioned at the middle indent of the jaws. The spheres and cylinders used in this experiment should be positioned similarly.

(d) Once the cylinder is placed between the jaws, be sure to tighten the locking screw. Record the measured quantity on the LCD display in the **diameter** column of the spreadsheet.

4. Follow the procedure outlined above and measure the diameter of your selected sphere.

5. Select another sphere and repeat this procedure until you have measured the mass and diameter of 5 spheres.

6. The **Density Lab** program is designed to calculate the volume and density of the chrome steel spheres using your data. Consult with your lab instructor if the program does not produce valid solutions with your data.

7. After recording all measurements for the spheres, select the **Graph of Data for Spheres** tab. The program produces a plot of the data you recorded.

8. **Print** one copy of the Data Sheet and the Graph of Data. Include these prints with your lab report.

9. Estimate and record the measurement uncertainty in the mass and diameter of the spheres. A good rule of thumb for doing this is to take the smallest value that an instrument can read and divide it by 2. For example, if the scale can read mass to $m = 0.1g$, then the uncertainty in a mass measurement using this scale is $\Delta m = \pm 0.05g$.

Density of the Cylinders

Be certain to convert all measurement to meters, kilograms, and seconds!!!

1. Select the tab entitled **Data Sheet for Cylinders**. All measurements performed using cylinders will be recorded on this spreadsheet.

2. Select one of the brass cylinders and measure its mass using the scale. Record this in the **mass** column of the spreadsheet.

3. Follow the procedures outlined for Using the Electronic Digital Caliper and measure the outer diameter of the cylinder. Record this value in the **diameter-out** column.

4. Measure the height of the cylinder and record this value in **length** column.

5. Insert the "beak" of the caliper (see Figure 1) into the hole of the brass cylinder. (Note: The jaws of the caliper should be closed before the "beak" of the caliper is inserted into the hole in the brass cylinder). Expand the jaws until the inner edges of the "beak" meet the edges of the hole. Measure the diameter of the hole and record this value in the **diameter-in** column.

6. Select another cylinder and repeat steps 2-5 until you have measured the mass and diameter of 4 cylinders.

7. The **Density Lab** program is designed to calculate the volume and density of the brass cylinders using your data. Consult with your lab instructor if the program does not produce valid solutions with your data.

8. After recording all measurements for the spheres, select the **Graph of Data for Cylinders** tab. The program produces a plot of the data you recorded.

9. **Print** one copy of the Data Sheet and the Graph of Data. Include these prints with your lab report.

10. Estimate and record the measurement uncertainty in the mass, height, and diameter of the cylinders.

Data Analysis

Density of Spheres

1. Calculate and record an average value of the density measurements.

2. Calculate the percent variation in the density values.

3. Compare the average density of the spheres to the density of chrome, which is $7.8 \times 10^3 kg/m^3$, by calculating the percent difference using your measured experimental value and the above-mentioned theoretical value.

4. Calculate the percent uncertainty in the mass of the spheres using the smallest measured value, the uncertainty value, and

$$\% \ uncertainty = 100 \times \frac{measurement \ uncertainty}{smallest \ measured \ value}. \tag{7}$$

5. Calculate the percent uncertainty in the diameter of the spheres using equation 7.

6. Using the values obtained for percent uncertainty in mass and diameter, determine the largest percent uncertainty in the experiment.

Density of the Cylinders

1. Calculate and record the average density values for the cylinders.

2. Calculate the percent variation in the density values.

3. Compare the average density of the cylinders to the density of brass, which is $8.5 \times 10^3 kg/m^3$, by calculating the percent difference using your measured experimental value and the above-mentioned theoretical value.

4. Calculate the percent uncertainty for mass measurements using the smallest measured value, the uncertainty value and

$$\% \ uncertainty = 100 \times \frac{measurement \ uncertainty}{smallest \ measured \ value}. \tag{8}$$

5. Calculate the percent uncertainty for measurements of height and diameter.

6. Using the values obtained for the percent uncertainty in mass, height and diameter determine the largest percent uncertainty in the experiment. **In your conclusion, be sure to compare the percent difference, percent variation and the largest percent uncertainty of the spheres independent of the percent difference, percent variation, and largest percent uncertainty of the cylinders.**

Final Comments

In the density of the cylinders experiment, the first calculation of volume disregarded the diameter of the hole, therefore these values are less than ideal. This would cause the density for the brass cylinders to be too small, since the density is the mass divided by the volume. Your analysis shows that the percent difference values for the density of copper using the first volume calculation is on the whole larger than the percent difference values for which the second volume calculation is used to determine density of copper. This kind of experimental uncertainty for which all data points are affected in the same way i.e. "pushed" in the same direction away from the theoretical value is known as *systematic error*.

Selected Questions

1. You and your lab partner each measure the diameter of a cylindrical piece of aluminum. You obtain a measurement of 8.15 *mm* and your partner obtains a measurement of 8.00 *mm*. Your partner says: *"Well, one of us definitely did something incorrectly"*. Do you agree or disagree? Explain your reasoning. [Based upon a problem from **Physics by Inquiry**, vol. 1, by Lillian McDermott p. 133 (1996)]

2. Suppose we have 68 *g* of aluminum. (The density of aluminum is 2.7 g/cm^3). How much more aluminum, in mass, is needed to make a total volume of 40 cm^3 . [From **Physics by Inquiry**, vol. 1, by Lillian McDermott p. 145 (1996)]

3. Suppose we have equal volumes of aluminum and a mystery metal. The mystery metal has a mass of 80 *g*. What is the density of the mystery metal if the aluminum has a mass of 19 *g* and a density of 2.7 g/cm^3 ? [From **Physics by Inquiry**, vol. 1, by Lillian McDermott p. 145 (1996)]