



Newton's Second Law

Objective

The Newton's Second Law experiment provides the student a hands on demonstration of "forces in motion". A formulated analysis of forces acting on a dynamics cart will be developed by the student. Students will calculate a theoretical acceleration value using their derived equation, and then compare their results to an experimental value.

Equipment List

PASCO Dynamics Cart with Track, Set of Hanging Masses, Two Photogates, PASCO Smart Timer, PASCO Photogate Sail, Table Clamp with Rod, Dynamics Track, Attachment to connect Rod to Track, Two Photogate Track Braces, Angle Indicator for Track, Ruler, Nylon String, Bubble Level, mass scale.

Theoretical Background

In order to get a stalled car moving, one must push or exert a force on the car. Since the car goes from a state of being at rest to a state of motion, the car must be accelerated, since acceleration is a change in velocity ($v_i = 0$; $v_f = v$ for an acceleration value a). The missing factor that relates the force to the acceleration can be deduced by considering that any one of us would prefer to push a VW Bug rather than an army Tank. The difference between the Bug and the Tank is, primarily, that the Tank is made of more "stuff" and is harder to get started. This resistance to motion, which is a measure of the amount of "stuff" something is made of, is known as mass. Mass, or inertial mass, is a measure of the resistance of an object to motion. This relationship was first postulated by Isaac Newton in his Second Law of Motion,

$$\Sigma F = ma. \quad (1)$$

In this equation, ΣF is the sum of the forces acting on an object, m is the mass of

the object, and a is the acceleration of the object. Both F and a denote vector quantities. The standard representation for vectors is the symbol such as F for force with a small directional arrow over it. To use this equation to calculate the acceleration, all of the forces acting on an object must be added, in a vector sense, to get the total force acting on the object.

Flat Track with Hanging Mass

In the first case of this lab exercise, a cart is attached by a piece of string to another mass which is hung over the table supporting the cart track by a pulley so that as the hanging mass falls, it pulls the cart along the track. For this kind of problem, it is useful to draw a diagram of the forces acting on each of the masses involved in the problem. The experimental setup is shown in Figure 1. The forces acting on the cart and hanging mass are labelled in Figure 1. Free-body diagrams of the forces on the cart and hanging mass are provided in Figure 2.

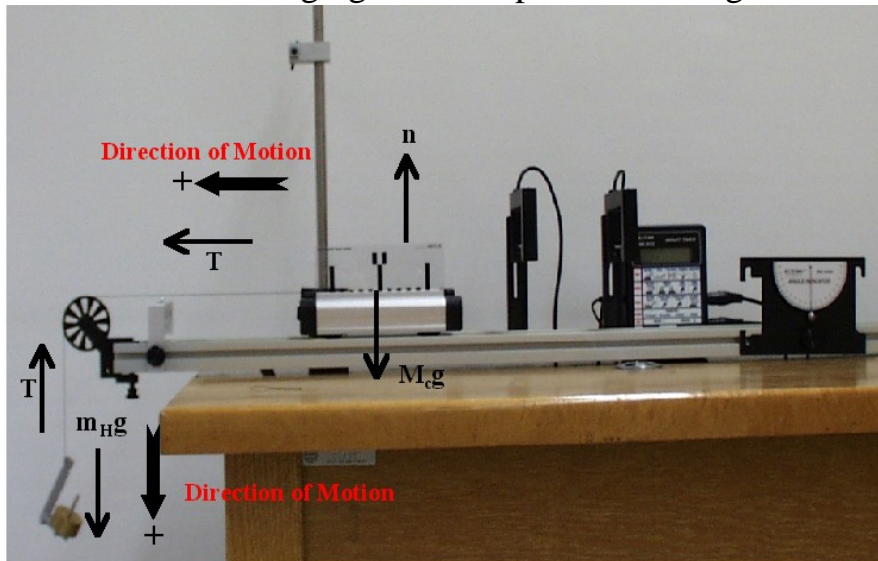


Figure 1: Forces on Dynamics Cart with Hanging Mass

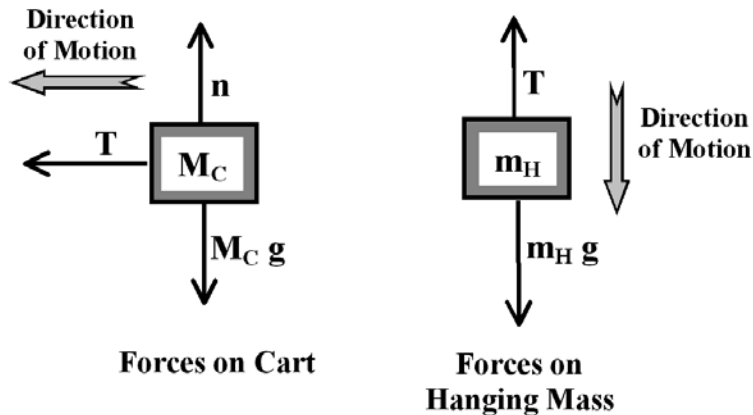


Figure 2: Free-Body Diagram of Forces on Cart and Hanging Mass

Since force is a vector quantity, it is necessary to decide what are the positive and negative directions so that each force can be labeled as being either positive or negative. A useful rule is to say that the direction of motion is the positive direction. This direction is indicated in both Figures 1 and 2. With this convention in place, equations for the total force acting on each object can be written down. For the cart, this equation is,

$$\Sigma F_{cart_x} = T = M_C a_C \quad (2)$$

In this equation, F_{cart_x} is the total force on the cart in the horizontal, or x-direction, T is the tension in the string, which always pulls away from the mass in the direction of the string, M_C is the mass of the dynamics cart, and a_C is the acceleration of the cart in the horizontal direction.

Since we do not expect the cart to lift off of the air track or collapse into the track, we can say that there is no net force acting in the vertical, or y, direction. This means that the normal force, n , must be balanced by the weight of the cart, $M_C g$, so that $n = M_C g$.

To solve for the acceleration of the cart, from Equation 2, we need to know the tension in the string, T . To obtain the tension in the string, consider the forces acting on the hanging mass. From the forces illustrated in Figure 2, the following equation can be written down using Newton's second law,

$$\Sigma F_H = m_H g - T = m_H a_H \quad (3)$$

In this equation, all of the variables have the same meaning with the addition that F_H is the total force on the hanging weight, m_H is the mass of the hanging weight, and a_H is the acceleration of the hanging weight.

Since both the hanging mass and the dynamics cart are connected by the same piece of string, which is assumed not to stretch, the same tension acts on both. This tension is responsible for accelerating the cart. For the tension to be the same on both the dynamics cart and on the hanging mass, the acceleration of each must also be the same. To demonstrate that this is correct, consider what would happen if the acceleration was *not* the same for both. For example, if the dynamics cart was to accelerate progressively faster than the hanging mass, then the string would go limp resulting in no tension in the string. If the hanging mass was to accelerate progressively faster than the dynamics cart, then the string

would snap maximizing the hanger's fall due to gravity. Since the string connecting the two neither loosens nor breaks, the tension acting on each object must be the same, and the acceleration of each mass must also be the same. This will be examined during the lab exercise. The variable for tension, T is used in both equations 2 and 3 indicating a certain understanding of tension acting on both the cart and hanging mass. Since the acceleration is the same for both, the two accelerations, a_c and a_H , can be replaced by a single variable for the acceleration, a . Equations 2 and 3 can be rewritten as,

$$M_c a = T, \quad (4)$$

$$m_H a = m_H g - T \quad (5)$$

Substituting the tension T in Equation 5 with the tension T from Equation 4, and solving for the acceleration (which is the same for both the hanging mass and the cart) gives,

$$a_{theo1} = \frac{m_H}{m_H + M_c} g \quad (6)$$

Inclined Track with Hanging Mass

In this case, a hanging mass attached to the cart on an inclined track is analyzed. Figure 3 illustrates the experimental situation and the forces acting on the cart and hanging mass. Free-body diagrams of the forces on the cart and hanging mass are shown in Figure 4.

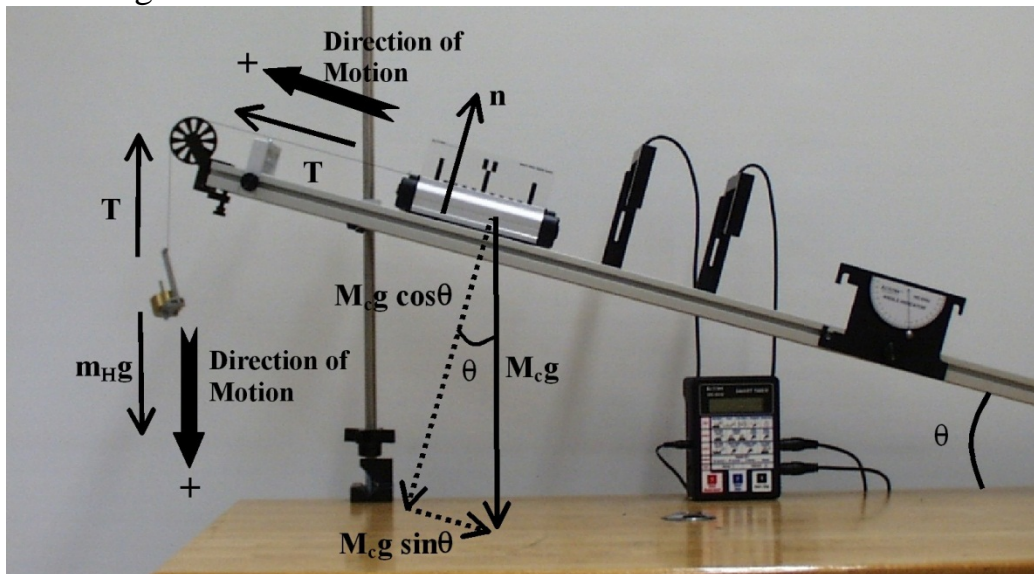


Figure 3: Inclined Track with Hanging Mass

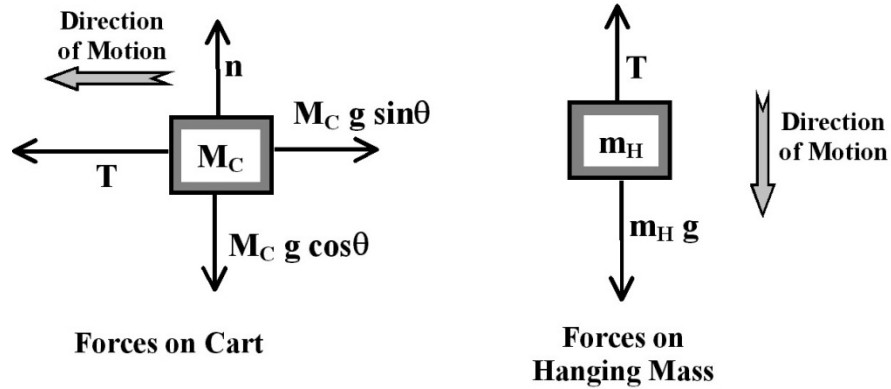


Figure 4: Free-Body Diagram of Forces on Cart and Hanging Mass

Adding the forces acting on the cart parallel to the track, as illustrated in Figure 4, and applying Newton's second law gives,

$$\Sigma F_{\parallel} = T - M_C g \sin \theta = M_C a_c. \quad (7)$$

In this case, the normal n is balanced by the component of the weight perpendicular to the track so that $n = M_C g \cos \theta$.

Adding the forces acting on the hanging mass and applying Newton's second law gives,

$$\Sigma F_H = m_H g - T = m_H a_H. \quad (8)$$

The magnitude of the acceleration of the hanging mass is the same as that for the cart since the hanging mass and the cart are attached by a string of constant tension, which is the same result stemming from the flat track case. Combining these two equations by eliminating the tension, and solving for the acceleration yields,

$$a_{theo2} = \frac{m_H g - M_C g \sin \theta}{m_H + M_C} \quad (9)$$

In this series of experiments, these theoretical relations for the acceleration will be tested experimentally.

Procedure

Flat Track with Hanging Mass

In this section, the acceleration of the cart on a flat track will be measured experimentally several times and averaged to obtain the average acceleration. The mass of the hanging weight will be varied, and the resulting acceleration measured repeatedly to obtain the average acceleration.

1. Use the mass scale to measure the mass of the dynamics cart and sail. Record this mass as M_C on your data sheet.
2. Place 10 g on the hanging mass holder so that the total mass of the hanging weight, including the holder, is 15 g. Measure and record the actual mass of the hanging weight and record this value on your data table.
3. Determine the theoretical value for the acceleration using the mass values measured from steps 1 and 2 and Equation 6. Record this acceleration in the $a_{theo\ 1}$ column of your data sheet.
4. Attach the hanging mass to the cart. **Make sure the string is long enough so that the entire cart successfully goes through the photogate before the hanging mass hits the floor.**
5. Make sure the track is not attached to the rod clamped to the table. If it is, remove it from the rod and lay it flat on the table. Place the cart on the track. Drape the hanging mass over the pulley. Adjust the height of the pulley so that the string is parallel to the track.
6. Use the bubble level to determine if the track is level. If the track is not level, adjust the small plastic screw at the end of the track to level the track. Consult with your lab instructor if difficulties arise during this process.
7. Adjust the photogates so that they are 30 cm apart. Adjust the heights of the photogates so that they are triggered by the same portion of the photogate sail established during the **Kinematics** experiment.
8. Set the Smart Timer to the **Accel: Two Gate** mode. Pull the cart back along the track 25 cm away from the closest photogate, and press the black key on the Smart Timer to ready the timer. An asterisk (*) should appear on the display of the Smart Timer under the **Accel: Two Gate**. **Do not press the black key until after you have pulled the cart through the photogate.**
9. Release the cart. The acceleration of the cart, in cm/s^2 , should appear on the Smart Timer. This value should be within 10 cm/s^2 of the theoretical value. If

it is not, have the lab instructor check your equipment. If it is, convert this to m/s^2 and record the value as a_{exp} on your data sheet.

10. Repeat steps 8 and 9 four times to obtain a total of five experimental values for the acceleration.
11. Repeat steps 2-10, for hanging mass values of $m_H = 25\text{ g}$, 35 g , and 45 g . For each value of the hanging weight, measure the acceleration of the cart five times. Be sure to measure the actual mass of the hanging weight for each trial.
12. For each value of the hanging mass, calculate the average value of the experimental acceleration. Record these average experimental accelerations in the $a_{exp,ave}$ column.
13. For each value of the hanging mass, calculate the percent variation in the experimental values of the acceleration, a_{exp} . Record these values in the space provided. **Record the largest percent variation in a_{exp} as the percent uncertainty in a_{exp} for this experiment.**
14. Estimate the uncertainty in the mass measurements.

Inclined Track with Hanging Mass

In this section, the acceleration of the cart on an inclined track will be measured experimentally. The mass of the hanging weight will be varied, and the resulting acceleration will be measured.

1. Attach the track to the rod clamped to the table. Incline the track to an angle of $\sim 15^\circ$ off the table. Determine the angle by measuring the height of the end of the track above the table, and apply the $\sin^{-1}(\theta)$ to the ratio of this height to the length of the track to find θ . Record the measured angle on your data sheet.
2. Place 160 g on the hanging mass, for a total hanging mass of 165 g including the mass of the hanger. Measure the mass of the hanging mass and record this mass on your data sheet.
3. Determine the theoretical value for the acceleration using the mass of the cart and the hanging weight mass value with Equation 9. Record this acceleration in the a_{theo} column of your data sheet.
4. Place the cart on the track. Adjust the height of the pulley so that the string is parallel to the track. Adjust the height of the photogates so that they are triggered by the correct portion of the sail.
5. Set the Smart Timer to the **Accel: Two Gate** mode. Pull the cart back along the track 25 cm away from the closest photogate, and press the black key on

the Smart Timer to ready the timer. An asterisk (*) should appear on the display of the Smart Timer under the **Accel: Two Gate** . **Do not press the black key until *after* you have pulled the cart through the photogate.**

6. Release the cart. The acceleration of the cart, in cm/s^2 , should appear on the Smart Timer. This value should be within 10 cm/s^2 of the theoretical value. If it is not, have the lab instructor check your equipment. If it is, convert this to m/s^2 and record the value as a_{exp} on your data sheet.
7. Repeat steps 5 and 6 four times to obtain a total of five experimental values for the acceleration.
8. Repeat steps 1-7 for hanging mass values of $m_H = 175\text{ g}$, 185 g , and 195 g . For each value of the hanging weight, measure the acceleration of the cart five times. Be sure to measure the actual mass of the hanging weight and record this mass on your data sheet.
9. For each value of the hanging mass, calculate the average value of the experimental acceleration. Record these average experimental accelerations in the $a_{exp,ave}$ column.
10. For each value of the hanging mass, calculate the percent variation in the experimental values of the acceleration, a_{exp} . Record these values in the space provided. **Record the largest percent variation in a_{exp} as the percent uncertainty in a_{exp} for this experiment.**
11. Estimate the uncertainty in the mass measurements.

Data Analysis

Flat Track with Hanging Mass

1. Calculate the percent difference between the theoretical and experimental values of the acceleration for each value of the hanging mass.
2. Plot the experimental value of the acceleration a_{exp} as a function of $\frac{m_H}{m_H + M_C}$. Fit the data with a straight line. Determine the slope and y-intercept of this line and record them on your data sheet.

Inclined Track with Hanging Mass

1. Calculate the percent difference between the theoretical and experimental values of the acceleration for each value of the hanging mass.

2. Plot the experimental value of the acceleration a_{exp} as a function of $\frac{m_H - M_c \sin \theta}{m_H + M_c}$. Fit the data with a straight line. Determine the slope and y-intercept of this line and record them on your data sheet.

Selected Questions

1. How would friction effect the experimental values of acceleration? Would friction be a source of systematic error or random error? Why? **Explain** your reasoning
2. If the pulley was not set so that the string was parallel to the track, what effect would this have on the acceleration of the system? What effect would this have on the normal force from the track? Would this be a random or systematic error? **Explain** your reasoning.
3. For the theoretical calculation of the acceleration, friction was ignored. Apply Newton's second law and derive an equation for the acceleration that would include a force of friction for the Flat Track case. Refer to equations 2-5 in the Theoretical Background section. Use your derived equation, and the first average experimental value of the acceleration for the Flat track case ($m_H = 15 \text{ g}$), to calculate the frictional force. How does this force of friction compare to the force pulling the cart along the track $m_H g$? What do your calculations of friction suggest about the assumption that friction is negligible? **Show your work!**
4. For the theoretical calculation of the acceleration for the Inclined Track case, friction was also ignored. Apply Newton's second law and derive an equation for the acceleration that would include a force of friction for the Inclined Track case. Refer to equations 7 and 8 in the Theoretical Background section. Use your derived equation, and the first average experimental value of the acceleration for the Inclined Track case ($m_H = 165 \text{ g}$), to calculate the frictional force. For the Inclined Track case, what was the force that was pulling the cart? How does the force of friction compare to the force pulling the cart along the track for the Inclined Track case? What do your calculations of friction suggest about the assumption that friction is negligible? **Show your work!**