

Study of Inductive and Capacitive Reactance and RLC Resonance

1 Objective

To understand how the reactance of inductors and capacitors change with frequency, and how the two can cancel each other to leave only ohmic resistance when the frequency of excitation assumes the resonance value.

2 Equipment List

An inductor ($L \approx 112 \text{ mH}$), a capacitor $C \approx 35 \mu\text{F}$, a resistor $R \approx 25 \Omega$, an oscilloscope, and a precision sine wave generator ($v = 4 \text{ V}$ peak-constant for $1 \text{ Hz} < f < 1000 \text{ Hz}$).

2.1 Description

This experiment is first concerned with the frequency dependence of the reactive component X_L of the impedance Z of an inductor L . Its influence will be studied by means of the circuit shown in Fig. 1.

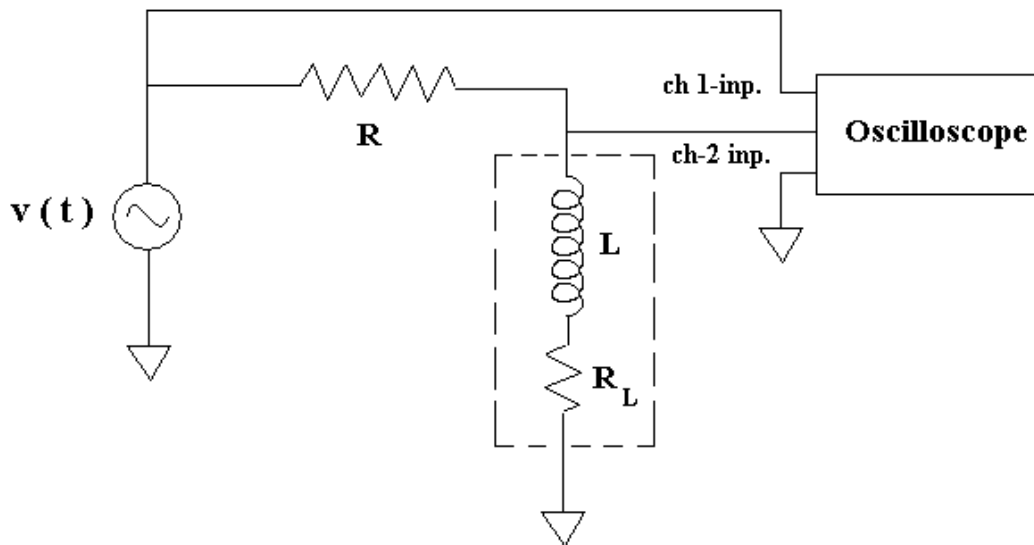


Figure 1. Schematic of the circuit used to study inductive reactance.

Inductive reactance

If the inductor had zero-internal resistance as opposed to its actual $R_L \approx 13 \Omega$ shown in the figure, then it would dissipate no heat even though the current $i(t)$ through it would be limited by the size of L in accord with the equation:

$$|v_L(t)| = (\omega L) i(t) \quad (1)$$

where $|v_L|$ is the magnitude of the sinusoidal voltage appearing across the ideal inductor, and $\omega = 2\pi f$.

The current through the inductor lags in phase behind the voltage across it by $\pi/2$ rad.

Capacitive reactance

In similar manner the voltage across a capacitor, which lags the a.c. current through it, is limited by the capacitive reactance $X_C = 1/(\omega C)$ according to the equation:

$$|v_C(t)| = i(t) / (\omega C) \quad (2)$$

The capacitive reactance will be studied using the circuit shown in Fig. 2.

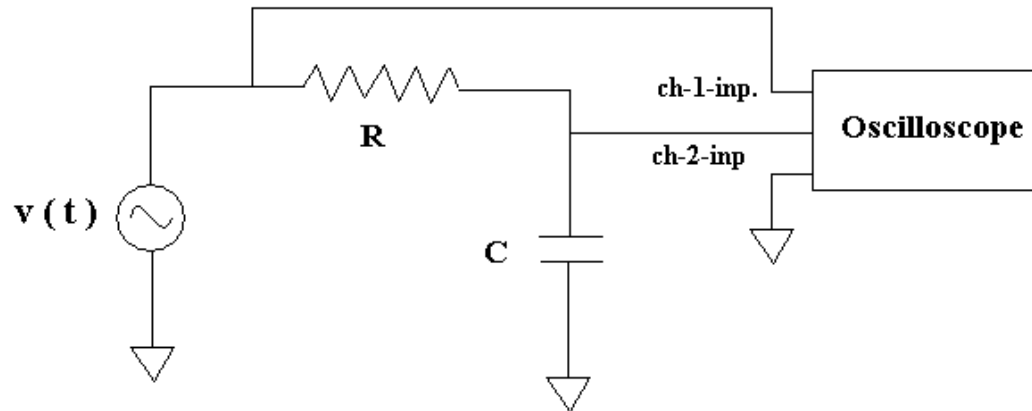


Figure 2. Schematic of the circuit used to study capacitive reactance.

Procedure

Inductor voltage vs frequency for an RL circuit

With the circuit of Fig. 1, using $R = 25\ \Omega$ set the voltage at a peak value of 4 V. The sine wave signal on ch-1 of the oscilloscope will then always graze the top and bottom reticules of the screen. This voltage will be maintained constant as the frequency is varied from its minimum value of 1 Hz to its maximum of 1000 Hz. For about one-dozen frequency settings in this range, measure the peak value of the voltage appearing across the inductor. This inductor voltage is read from the ch2 trace of the oscilloscope.

After recording the f / v_L pair of values, put the results into an Excel spreadsheet and generate a graph of the inductor voltage, such as shown in the upper left graph of Fig. 3. Observe that a log-scale is used for the frequency, to accommodate the large (three decade) range.

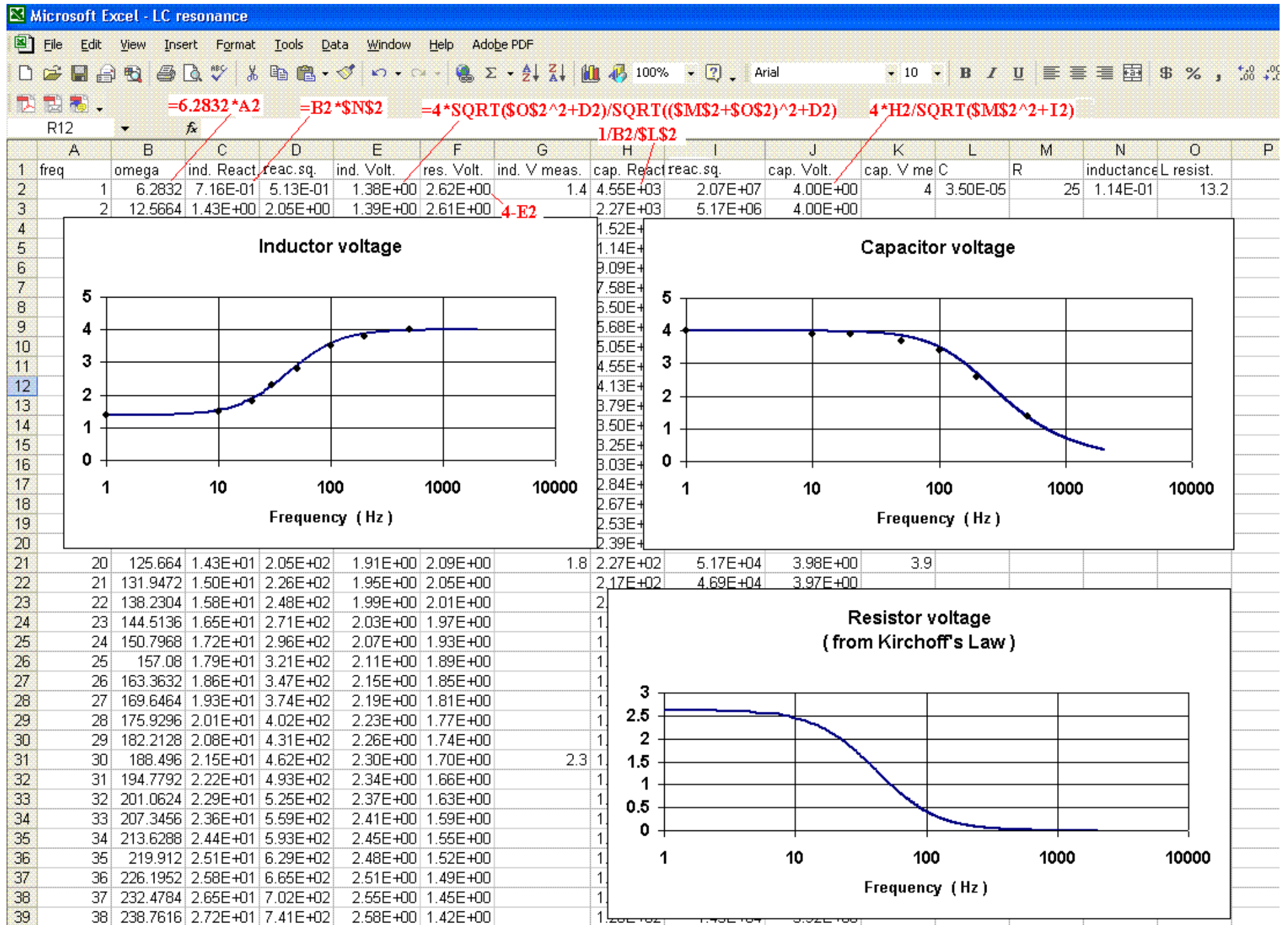


Figure 3. Screen print copy of an Excel spreadsheet used to graph RLC voltages.

Notice that a theoretical fit (smooth curve) has been applied to the inductor voltage set of points. That curve is governed by

$$v_L = \frac{\sqrt{R_L^2 + \omega^2 L^2}}{\sqrt{(R + R_L)^2 + \omega^2 L^2}} v \quad (3)$$

Using Eq.(3) supply a 'best fit' theory to your experimental data by adjusting the value of L. Before doing this adjustment, measure all of the parameters except L using the multimeter, and enter your measured values into your spreadsheet.

Capacitor voltage vs frequency for an RC circuit

Using the circuit of Fig. 2, repeat the procedure that was just finished with the inductor circuit. Once the data has been put into Excel and graphed, you should see a curve like the one shown in the upper right graph of Fig. 3.

A theoretical curve has also been fit to the capacitor voltage vs frequency graph. The equation that was used is

$$v_c = \frac{\frac{1}{\omega C}}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} v \quad (4)$$

Supply the same fit to your data using this equation.

QUESTION:

Which circuit (Fig. 1 or Fig. 2) was used to produce the lowest (third) graph of Fig. 3 ? Generate the equivalent to this graph using your data.

Resonance

Because the inductive reactance is proportional to frequency, whereas the capacitive reactance is inversely proportional to frequency, the two can cancel each other when connected as in Fig. 4.

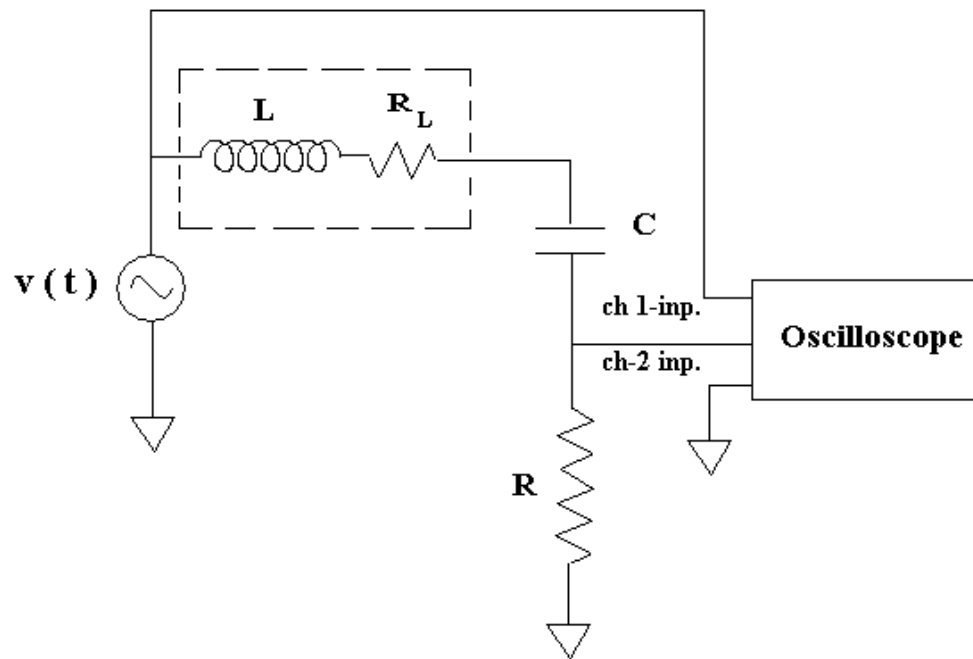


Figure 4. Circuit with which to study resonance.

The resonance frequency is given by

$$f_{\text{res}} = \frac{1}{2\pi \sqrt{LC}} \quad (5)$$

For the constant 4 V drive signal, set the frequency to several values that allow a sweep through resonance, to generate an Excel graph such as the one shown in Fig. 5.

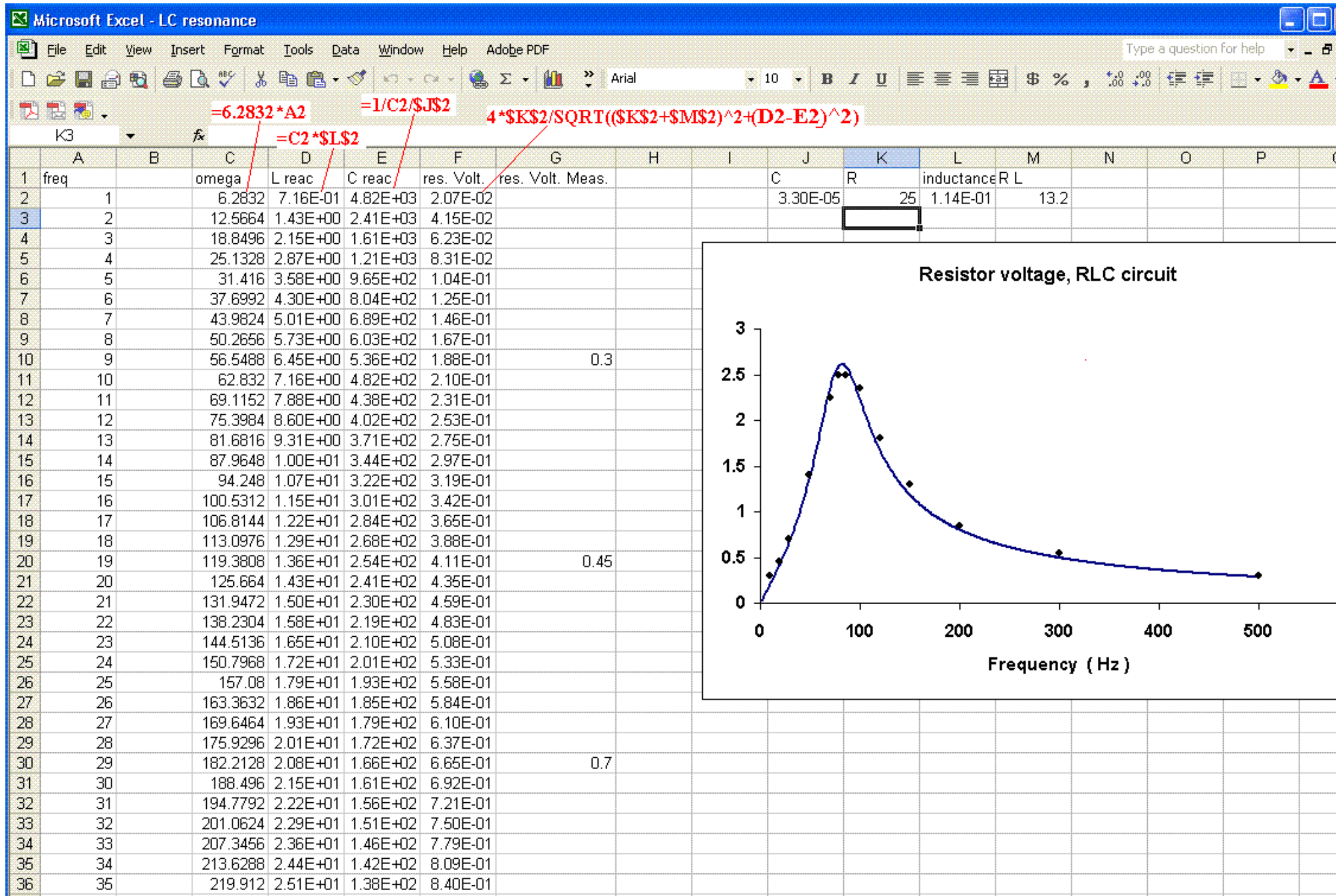


Figure 5. Screen print copy of an Excel spreadsheet used to graph the resistor voltage of Fig. 4 as the drive frequency is swept through resonance.

Observe that a theoretical fit has been supplied to the resonance curve; the equation being

$$v_R = \frac{R}{\sqrt{(R + R_L)^2 + (\omega L - \frac{1}{\omega C})^2}} v \quad (6)$$

Supply the same fit to your data.

Repeat this resonance study using $R = 5 \Omega$. How does the width of the resonance curve compare with the first case? (Disregard the fact that the peak voltage at resonance is lower.) The width is inversely proportional to the quality factor Q . Which case has the higher Q ?

Phase

To this point you have looked only at how the magnitude of the resistor voltage changes as a function of frequency. Now the oscilloscope will be used to look at the phase relationship between the drive voltage and the resistor voltage as one tunes through resonance.

Referring to Fig. 4, connect the 'high side' terminal of the oscillator to the unused channel of the oscilloscope. First, as a function of time, look at the displayed signal pair as the frequency is swept from below to above resonance. Take note of which voltage is lagging or leading.

Now change the mode of operation of the oscilloscope from a time display to an x-y display. Repeat the process of sweeping through resonance. Take note of the shape of the displayed ellipse and how its eccentricity changes with frequency. At what frequency does the ellipse become degenerate; i.e., a straight line?

Change the oscillator waveform from a sinusoidal drive to a square wave drive and sweep through resonance. Look at the signals using both the time-display and the x-y display. In the time-display case is there anything especially noteworthy about the shape of the resistor voltage when the frequency corresponds to resonance? On a piece of paper sketch this case.

The phase of the resistor voltage changes, relative to the drive voltage, as a function of frequency. Theoretically, the variation is as shown in Fig. 6; the graph was generated using

$$\phi = -\tan^{-1} \frac{\omega L - \frac{1}{\omega C}}{R + R_L} \quad (7)$$

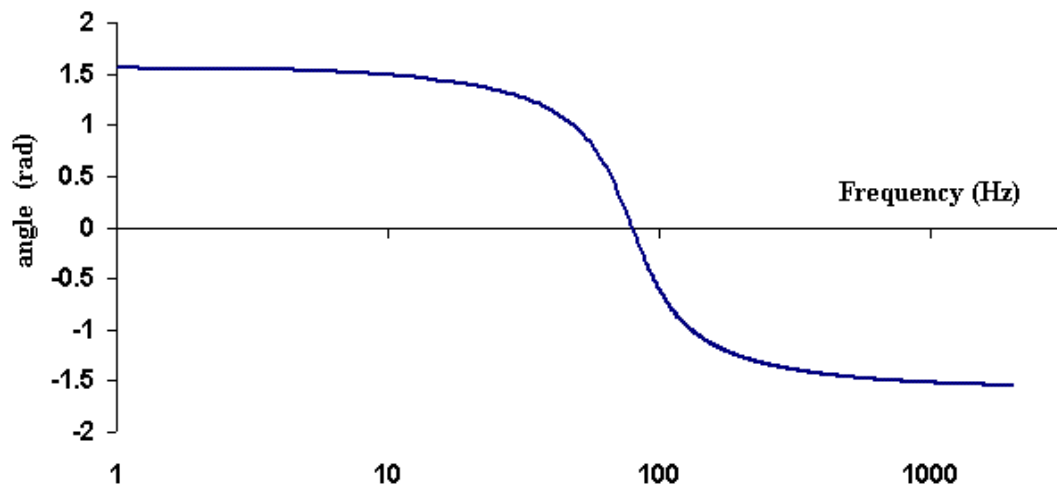


Figure 6. Plot showing how the phase changes as the circuit is tuned through resonance.

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