



RC Circuits and The Oscilloscope

Physics Lab X

Objective

In this series of experiments, the time constant of an RC circuit will be measured experimentally and compared with the theoretical expression for it. The rules for series and parallel combinations of resistors and capacitors will also be explored. Students will also become familiar with using the oscilloscope to make voltage measurements.

Equipment List

Tektronix TDS-2012 Digital Oscilloscope, Two $10\text{ k}\Omega$ Resistors, One $15\text{ k}\Omega$ Resistor, Two $0.1\ \mu\text{F}$ Capacitors, PASCO Digital Function Generator - Amplifier, D-cell Battery, Multimeter, BNC Adapter, Banana Plug Leads, Alligator Clips.

Theoretical Background

In a previous experiment, the properties of series and parallel combinations of resistors were explored. In particular, resistors in series added directly,

$$R_T = R_1 + R_2 + \dots, \quad (1)$$

while, for combinations of resistors in parallel, the reciprocals added,

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \dots \quad (2)$$

In the above equations, R_T is the total resistance of the combination, R_1 is the resistance of the first resistor, and R_2 is the resistance of the second resistor.

Capacitors have similar combination relations. Capacitors in parallel combine like resistors in series,

$$C_T = C_1 + C_2 + \dots \quad (3)$$

Capacitors in series combine like resistors in parallel,

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} \dots \quad (4)$$

However, the rules for voltage and current division are the same for both: For resistors and capacitors in parallel, the voltage drop across each is the same. For resistors and capacitors in series, the sum of the voltage drops across each is equal to the total voltage drop. The current, or charge in the case of capacitors, is the same for resistors and capacitors in series, while in parallel, the current through, or charge on, each is equal to the total current, or charge.

If a circuit is composed of both resistors and capacitors, the current flowing in the circuit and the charge on the capacitors no longer remains independent of time. There are two cases which are particularly interesting. In each case, a capacitor is connected in series with a resistor. For circuits containing more than one of each, the rules outlined above can be applied to reduce the combinations to a single equivalent resistor and a single equivalent capacitor.

In the first case, a resistor is connected to a capacitor (initially uncharged), a battery, and a switch which is initially open but closes at the beginning of the experiment, all in series. Figure 1 is a circuit diagram of this case.

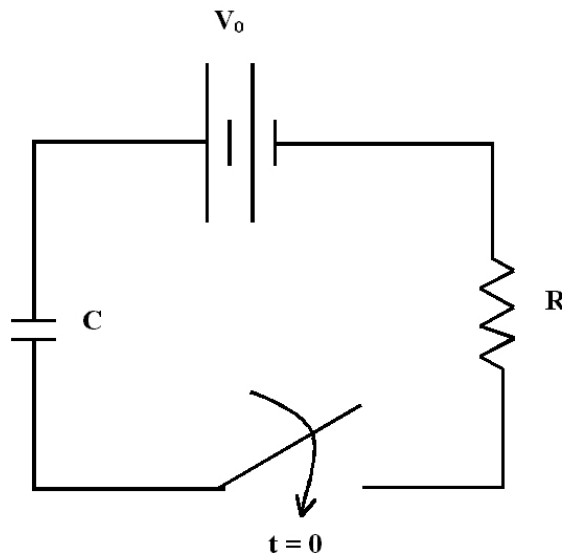


Figure 1: Charging RC Circuit Diagram

In this case, when the switch closes, current flows through the circuit causing the capacitor to gradually charge. As the capacitor charges, it opposes the flow of current causing the current to decrease. The buildup of charge causes the voltage across the capacitor to increase while the voltage across the resistor decreases and the current

decreases. This process can be represented mathematically by the following equations:

$$Q(t) = CV_0(1 - e^{-t/\tau}) \quad (5)$$

$$I(t) = \frac{V_0}{R}e^{-t/\tau} \quad (6)$$

$$V_C(t) = V_0(1 - e^{-t/\tau}) \quad (7)$$

$$V_R(t) = V_0e^{-t/\tau} \quad (8)$$

$$(9)$$

In these equations, Q is the charge on the capacitor as a function of time, C is the capacitance of the capacitor, t is the time increment, I is the current in the circuit, V_C is the voltage across the capacitor, and V_R is the voltage across the resistor. The variable τ is the time constant of the circuit. It governs the rate for which things can happen in the circuit. If τ is small, things happen quickly in the circuit, meaning voltage builds up quickly on the capacitor and the current falls rapidly. Mathematically, the time constant is the product of the value of the resistance and the capacitance,

$$\tau = RC. \quad (10)$$

The experimental case in which the capacitor is charging in the circuit will be referred to as the charging capacitor case.

The other case of interest is a capacitor, initially charged, is connected in series to a resistor and a switch which is initially open. A circuit diagram of this case is shown in Figure 2.

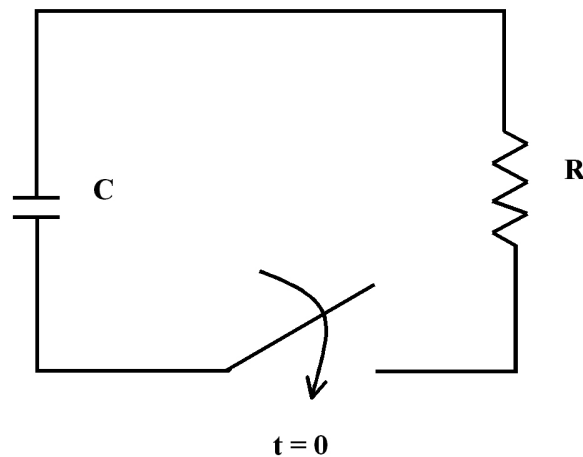


Figure 2: Discharging Capacitor Case Circuit Diagram

In this case, when the switch is closed, the capacitor discharges causing current to flow in the circuit. The energy stored in the capacitor is dissipated by the heating of the resistor. The voltage, current, and charge dissipate exponentially in time. These processes are represented mathematically by the following set of equations,

$$Q(t) = CV_0e^{-t/\tau} \quad (11)$$

$$I(t) = \frac{V_0}{R}e^{-t/\tau} \quad (12)$$

$$V_C(t) = V_0e^{-t/\tau} \quad (13)$$

$$V_R(t) = V_0e^{-t/\tau}. \quad (14)$$

$$(15)$$

The same time constant discussed earlier also controls the rate of these processes. Again, the experimental case for which the capacitor is discharging in the circuit will be referred to as the discharging case.

In this series of experiments, the time constant τ for a discharging RC circuit will be measured using an oscilloscope. To do this, note that, from Equation 13, the voltage across the capacitor is equal to e^{-1} when the time is equal to the time constant. Numerically, e^{-1} can be approximated, to within a 2% difference, by the fraction $\frac{3}{8}$,

$$V_C(t) = V_0e^{-t/\tau} \longrightarrow \frac{V_C(t = \tau)}{V_0} = e^{-1} \approx \frac{3}{8}. \quad (16)$$

In other words, when a time interval equaling the time constant has passed, the voltage across the capacitor is $\frac{3}{8}$ of the initial voltage. The oscilloscope will be used to measure how long it takes for the voltage to fall to this fraction of the initial voltage. The time constant for various circuit combinations of resistor and capacitors will be measured experimentally. A comparison between theoretical and experimental values of the time constant will be determined after recording appropriate measurements of the analyzed circuits.

Procedure and Data Analysis

Oscilloscope Exercises

In this series of exercises, various simple measurements will be made in order for you to get acquainted with the oscilloscope.

Measurement of Battery Voltage

In this section, you will learn to read and adjust the vertical scale by determining the voltage of a battery.

1. Attach the BNC adapter to the CH 1 terminal of the oscilloscope.
2. Insert leads into the BNC adapter. Use leads that have the same colors as the terminals. A small notch on the BNC adapter indicates black as ground. Attach alligator clips to the leads.

3. Switch the unit “ON.” The switch is located on top of the unit. Allow the unit to “boot up.” This operation will last less than a minute.
4. Press the yellow “CH 1 Menu” button. Perform the following operations if necessary: change COUPLING to “Ground;” change BW LIMIT to “Off;” change VOLTS/DIV to “Coarse;” change PROBE to “1X;” change INVERT to “off.” We will refer to this procedure as **Grounding** the oscilloscope.
5. Set the **VOLTS/DIV** for CH 1 to 1.00 V (the read-out for the VOLTS/DIV setting is located on lower left corner of the display). Set the **SEC/DIV** for CH 1 to 1.00 s.
6. Notice that the *trace* of the oscilloscope (yellow line) represents the voltage across the terminals as a function of time. Therefore the vertical axis represents voltage and the horizontal axis represents time.
7. The VERTICAL POSITION of the trace is established by adjusting the associated knob. Be sure the vertical position of the trace is set to 0.00 *divs*. Horizontal position adjustments in the Scan Mode are inactive.
8. Connect the alligator clips to the red and black terminals of the battery holder. Change COUPLING to “DC.”
9. Press the “Measure” button. Perform the following operations for Measure 1: change SOURCE to “CH1;” change TYPE to “Mean.” Press BACK. Press the black “Auto Set” button. Obtain a measurement for the mean voltage value of the battery.
10. Use the multimeter to determine the voltage of the battery. The voltage measured using the multimeter should be comparable to the mean voltage value measured from the oscilloscope. If this is not the case consult with your lab instructor.

Period of Sinusoidal Voltage Measurement

In this section, students will learn how to make time measurements using the oscilloscope by measuring the period of a sinusoidal voltage source.

1. If necessary, **ground** the oscilloscope.
2. Remove the alligator clips from the leads. Connect the leads from the oscilloscope to the Function Generator. Use leads that have the same colors as the terminals
3. Switch the Function Generator “ON.” The switch is located on the back of the unit. Change the “RANGE” setting of the function generator to 100.00 Hz . Note, the frequency displayed is the frequency of the output voltage.
4. On the oscilloscope change the COUPLING to “DC.” Press the “Measure” button. Perform the following operations: for Measure 1 change SOURCE to “CH1;” change TYPE to “Freq;” select “Back” and for Measure 2 change TYPE to “Period;” select “Back” and for measure 3 change TYPE to “Pk-Pk.” Select “Back.”
5. Adjust the “AMPLITUDE” control on the Function Generator so that the “Pk-Pk” measurement is between 2.50 V and 3.50 V .
6. Create a standing wave by adjusting the **SEC/DIV** control. It may be necessary to change the “ADJUST” control in order to obtain the desired standing wave pattern.
7. Record the “FREQUENCY” value from the Function Generator on your data sheet. Calculate the period from this value using the equation and record the result on your data sheet.

$$T = \frac{1}{f} \quad (17)$$

8. Record the period of oscillation value from the oscilloscope on your data sheet.
9. Adjust the frequency of the Function Generator to 2000 Hz . Repeat steps 1 through 8.
10. Adjust the frequency of the Function Generator to 50000 Hz and repeat steps 1 through 8. Ground the oscilloscope and set the “AMPLITUDE” of the Function Generator to MIN after completing this exercise.

RC Time Constants

In this section, the oscilloscope will be used to measure the time constant of RC circuits; circuits containing *resistors* and *capacitors* are used in series and parallel arrangement to measure time constants.

Individual Resistors and Capacitors

1. Use a multimeter to measure the values of a $10\text{ k}\Omega$ resistor and a $0.1\ \mu\text{F}$ capacitor. The color code values are *NOT* significant for this experiment. Record the measured values on your data table.
2. Combine the $10\text{ k}\Omega$ resistor and the $0.1\ \mu\text{F}$ capacitor in a series circuit with the function generator. Refer to the circuit diagram in Figure 3. Connect the oscilloscope so that it measures the voltage drop across the capacitor. ***Be careful that the Function Generator ground is connected to the Oscilloscope ground.***

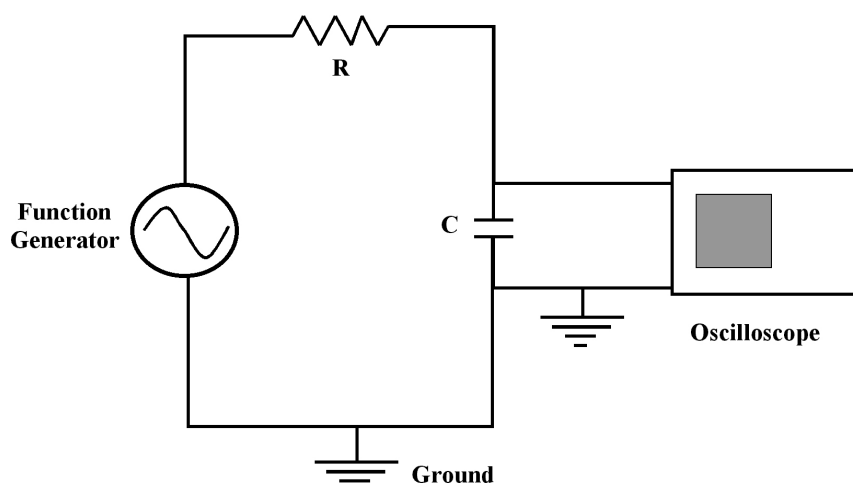


Figure 3: Circuit Diagram for RC Circuit with Oscilloscope

3. Change the "ADJUST" control to 100 Hz . Change the "WAVEFORM" selection to the square wave pattern.

4. Change the COUPLING to “DC.” Press the “Measure” button. The measurement selections should be the same selections from the Period of Sinusoidal Voltage Measurement exercise.
5. Adjust the “AMPLITUDE” control on the Function Generator so that the “Pk-Pk” measurement is between 1.95 V and 2.00 V.
6. Create a standing wave pattern by adjusting the SEC/DIV control. Refer to Figure 4.

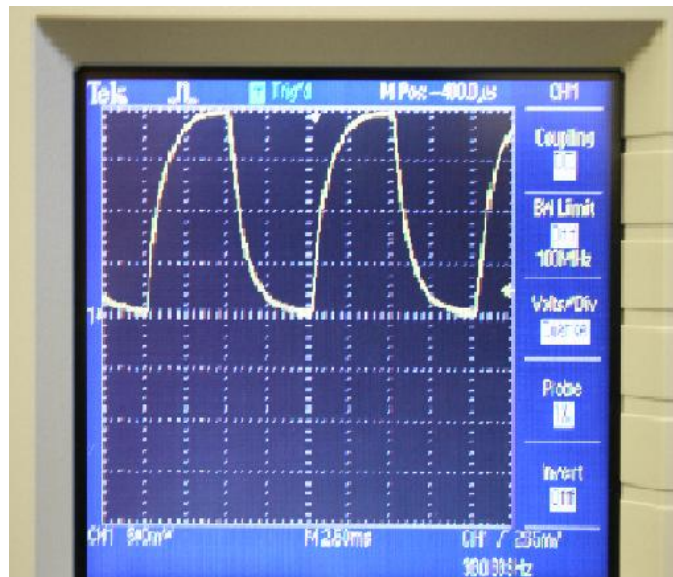


Figure 4: Trace of Voltage Drop Across Capacitor of RC Circuit

7. Adjust the “HORIZONTAL POSITION” control on the oscilloscope so that the cycle begins at an initial time of zero. See Figure 5.(note, time is measured along the horizontal).
8. Notice that the voltage across the capacitor decays through four units along the vertical; each of the four units can be divided into five divisions for a total of twenty divisions. The value at 7.5 divisions (along the vertical) marks the point where the voltage across the capacitor is $\frac{3}{8}$ of the initial voltage.

$$\frac{3}{8}(20 \text{ divisions}) = 7.5 \text{ divisions} \quad (18)$$

Measure and record the time constant τ corresponding to the initial voltage at 7.5 divisions. You will first need to determine the time-scale. This value is centered at the bottom of the display. The experimental time constant is determined from the number of divisions along the horizontal.

$$\tau_{exp} = \frac{\text{number of divisions}}{5} (\text{time scale value}) \quad (19)$$

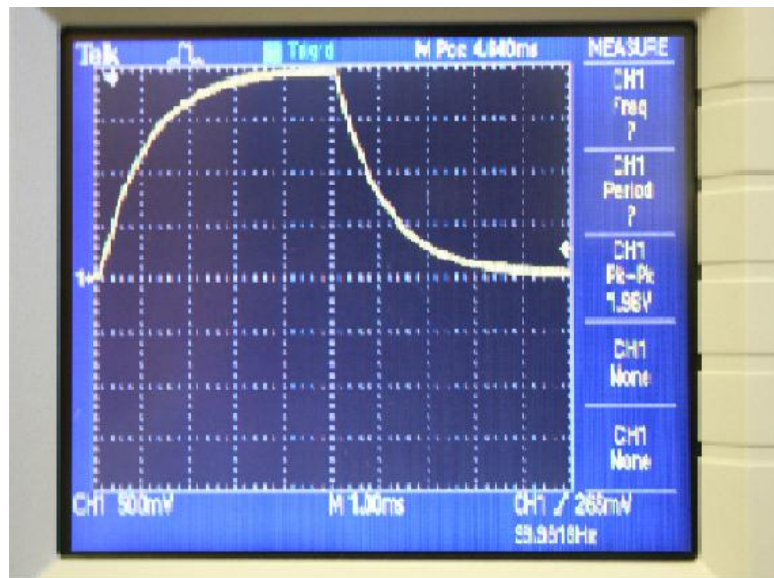


Figure 5: Trace Position and Time Constant Measurement from Oscilloscope

9. Calculate the theoretical value of the time constant using measured values of resistance and capacitance.

$$\tau_{theo} = RC. \quad (20)$$

10. Calculate the percent difference between the experimental and theoretical values for the time constant.
11. The measured uncertainty for the time constant, τ_{exp} is the following:

$$\text{measured uncertainty} = \frac{\text{time scale value}}{10}. \quad (21)$$

Record this value on your data table.

12. Use the measured uncertainty value along with the measured value of the time constant τ_{exp} to calculate the percent uncertainty of time constant measurements.

$$\% \text{ Uncertainty} = 100 \times \frac{\text{Uncertainty}}{\text{Measured Value}} \quad (22)$$

13. Ground the oscilloscope. Repeat the circuit set-up using the $15 \text{ k}\Omega$ resistor. Be sure to measure the value of the resistor.
14. Estimate and record the uncertainty of the resistors used and the capacitor used. Determine the smallest measured values and calculate the percent uncertainty.

Two Resistors in Series/Single Capacitor

1. Use the multimeter to measure the resistance of the $10 \text{ k}\Omega$ resistors and the capacitance of the $0.1 \mu\text{F}$ capacitor. Record these values on your data table.
2. Construct a circuit with two $10 \text{ k}\Omega$ resistors in series and one $0.1 \mu\text{F}$. Refer to Figure 6.

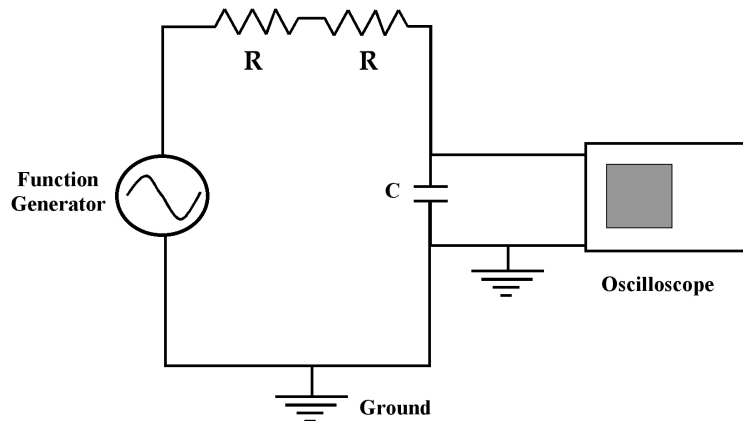


Figure 6: Circuit Diagram for RC Circuit with Two Resistors in Series

3. Repeat the procedure from the Individual Resistors and Capacitors exercise to make measurements of the time constant for this circuit.
4. Using the rules for combining resistors in series, calculate the total resistance of the RC circuit. Calculate the time constant of the RC circuit. Calculate the percent difference between the experimental and theoretical values of the time constant.
5. Estimate the uncertainty in the experimental determination of the time constant, τ_{exp} . (See the **Individual Resistors and Capacitors** subsection for further instruction.) Record the uncertainty value as τ_{exp} on your data table.
6. Use the uncertainty value along with the measured value of the time constant, τ_{exp} to calculate the percent uncertainty of time constant measurements.

$$\% \text{ Uncertainty} = 100 \times \frac{\text{Uncertainty}}{\text{Measured Value}} \quad (23)$$

7. Estimate and record the uncertainty of the resistors and the capacitor used for this circuit. Determine the smallest measured values and calculate the percent uncertainty.

Two Resistors in Parallel/Single Capacitor

1. Construct a circuit with two $10 \text{ k}\Omega$ resistors in parallel and one $0.1 \mu\text{F}$. Refer to Figure 7.
2. Repeat the procedure from the Individual Resistors and Capacitors exercise to make measurements of the time constant for this circuit.
3. Using the rules for combining resistors in parallel, calculate the total resistance of the RC circuit. Calculate the time constant of the RC circuit. Calculate the percent difference between the experimental and theoretical values of the time constant.

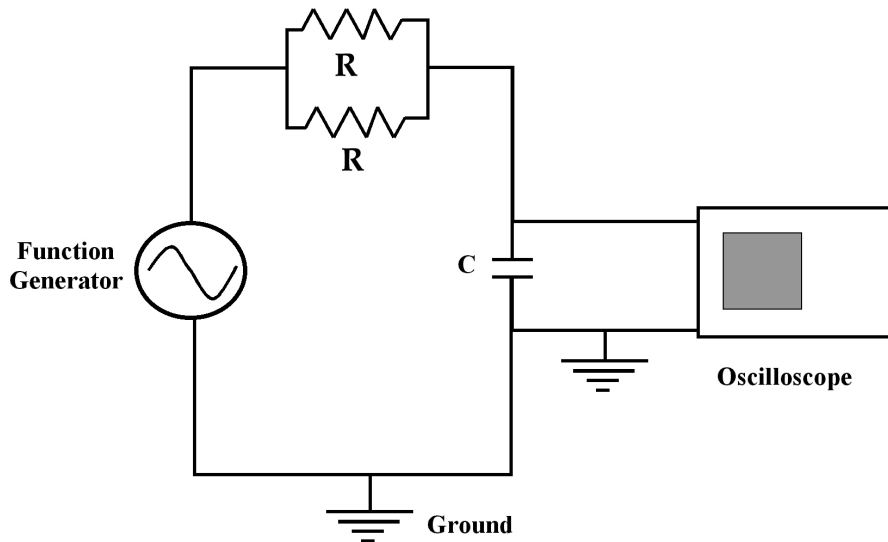


Figure 7: Circuit Diagram for RC Circuit with Two Resistors in Series

4. Estimate the uncertainty in the experimental determination of the time constant, τ_{exp} . (See the **Individual Resistors and Capacitors** subsection for further instruction.) Record the uncertainty value as τ_{exp} on your data table.
5. Use the uncertainty value along with the measured value of the time constant, τ_{exp} to calculate the percent uncertainty of time constant measurements.

$$\% \text{ Uncertainty} = 100 \times \frac{\text{Uncertainty}}{\text{Measured Value}} \quad (24)$$

6. Estimate and record the uncertainty of the resistors used and the capacitor used. Determine the smallest measured values and calculate the percent uncertainty.

Single Resistor/Two Capacitors in Series

1. Construct a circuit with one $10 \text{ k}\Omega$ resistor and two $0.1 \mu\text{F}$ capacitors in series. Refer to Figure 8. Notice that the Oscilloscope measures the voltage drop across *both* capacitors. Record the measured values of resistor and the two capacitors on your data table.
2. Repeat the procedure from the Individual Resistors and Capacitors exercise to make measurements of the time constant for this circuit.
3. Using the rules for combining capacitors in series, calculate the total capacitance of the RC circuit. Calculate the time constant of the RC circuit. Calculate the percent difference between the experimental and theoretical values of the time constant.
4. Estimate the uncertainty in the experimental determination of the time constant, τ_{exp} . (See the **Individual Resistors and Capacitors** subsection for further instruction.) Record the uncertainty value as τ_{exp} on your data table.

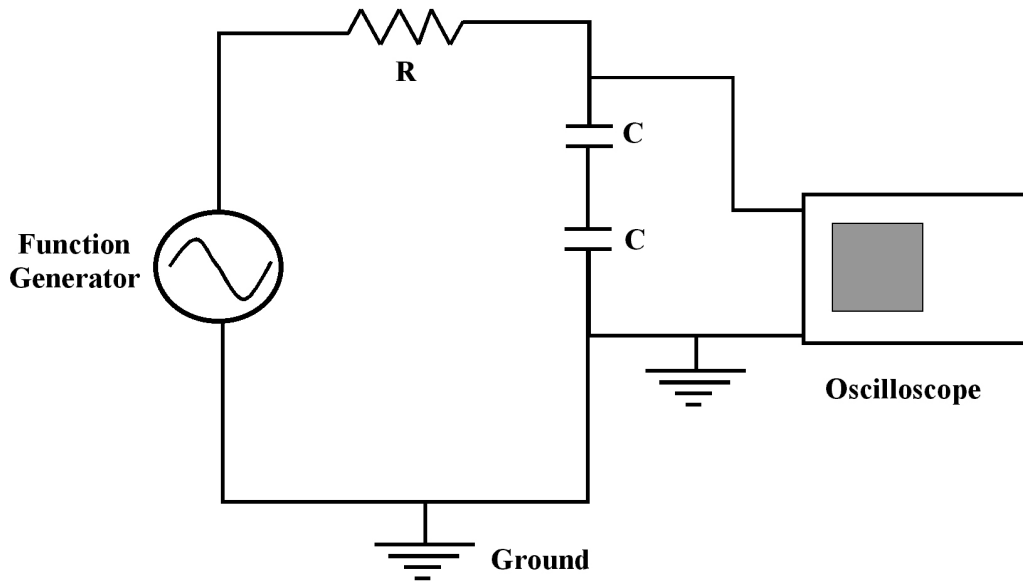


Figure 8: Circuit Diagram for RC Circuit with Two Capacitors in Series

5. Use the uncertainty value along with the measured value of the time constant, τ_{exp} to calculate the percent uncertainty of time constant measurements.

$$\% \text{ Uncertainty} = 100 \times \frac{\text{Uncertainty}}{\text{Measured Value}} \quad (25)$$

6. Estimate and record the uncertainty of the resistor used and the capacitors used. Determine the smallest measured values and calculate the percent uncertainty.

Single Resistor/Two Capacitors in Parallel

1. Construct a circuit with one $10 \text{ k}\Omega$ resistor and two $0.1 \mu\text{F}$ capacitors in parallel. Refer to Figure 9. Notice that the Oscilloscope measures the voltage drop across *both* capacitors. Record the measured values of the capacitors on your data table.
2. Repeat the procedure from the Individual Resistors and Capacitors exercise to make measurements of the time constant for this circuit.
3. Using the rules for combining capacitors in parallel, calculate the total capacitance of the RC circuit. Calculate the time constant of the RC circuit. Calculate the percent difference between the experimental and theoretical values of the time constant.
4. Estimate the uncertainty in the experimental determination of the time constant, τ_{exp} . (See the **Individual Resistors and Capacitors** subsection for further instruction.) Record the uncertainty value as τ_{exp} on your data table.
5. Use the uncertainty value along with the measured value of the time constant, τ_{exp} to calculate the percent uncertainty of time constant measurements.

$$\% \text{ Uncertainty} = 100 \times \frac{\text{Uncertainty}}{\text{Measured Value}} \quad (26)$$

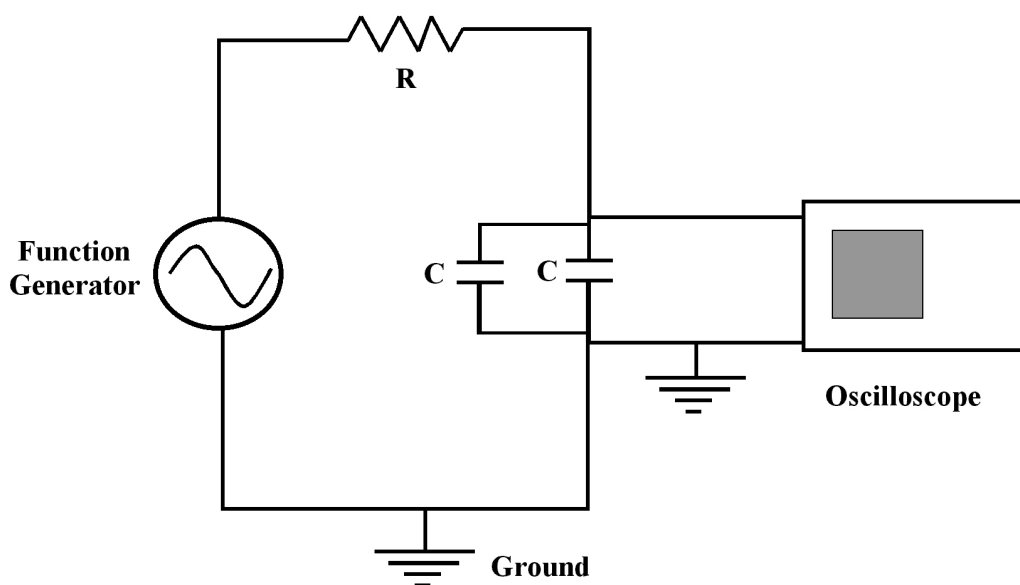


Figure 9: Circuit Diagram for RC Circuit with Two Capacitors in Parallel

6. Estimate and record the uncertainty of the resistor used and the capacitors used. Determine the smallest measured values and calculate the percent uncertainty.

Selected Questions

1. What effect on the total resistance of the circuit did adding an resistor in series have? How did this effect the time constant of the circuit?
2. What effect on the total resistance of the circuit did adding an resistor in parallel have? How did this effect the time constant?
3. What effect on the total capacitance of the circuit did adding an capacitor in series have? How did this effect the time constant?
4. What effect on the total capacitance of the circuit did adding an capacitor in parallel have? How did this effect the time constant?
5. What effect should the approximation of e^{-1} as the fraction $\frac{3}{8}$ have on your results? Do your results show this effect? Is this effect a source of random or systematic error?
6. Would you be justified in using a square wave frequency of 10 Hz when measuring a time constant of 0.02 s ? Why or why not? Could you use a square wave frequency of 100 Hz to measure a time constant of 0.02 s ? Why or why not?