Kirchhoff's Laws
Physics Lab IX

Objective

In the set of experiments, the theoretical relationships between the voltages and the currents in circuits containing several batteries and resistors in a network, known as Kirchhoff's laws, will be tested experimentally. In addition, the current within a network of nodes will be calculated first using Kirchhoff's Laws, and then measured experimentally.

Equipment List

4 'D' Batteries, 3 Resistors with resistance 50 $k\Omega$, 20 $k\Omega$ and 10 $k\Omega$, One Multimeter, Two Battery Holders for a Single 'D' Battery, Battery Holder for Two 'D' Batteries, 8 - 10 Connecting Leads, Alligator clips.

Theoretical Background

In previous lab experiments, the total resistance of a combination of resistors was calculated using the formulas for series and parallel combinations of resistors. Relationships between the current and the voltage drop across each resistor was also tested. In these circuits, a single battery was present. If a circuit contains more than one battery, these relationships could not be employed. Instead, another set of relationships, known as *Kirchhoff's Laws*, would be used. These relationships are named in honor of the German physicist Gustav Kirchhoff who discovered them.

Kirchhoff's first law states that the sum of the currents flowing through a node must be zero. This law is particularly useful when applied at a position where the current is split into pieces by several wires. The point in the circuit where the current splits is known as a *node*. Figure **??** illustrates a node in a typical circuit.
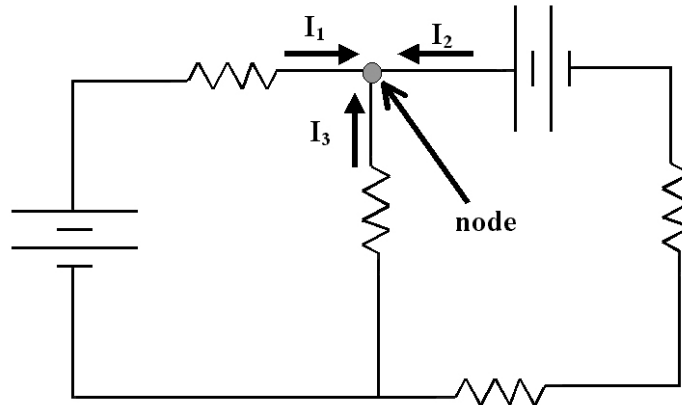
Figure 1: Current node in a circuit network

This relationship can be expressed mathematically

$$\sum_{i=1}^{N} I_i = I_1 + I_2 + I_3 + \ldots = 0, \tag{1}$$

where I_1, I_2, $I_3, \ldots$ are the current through each wire that branches from the node. For Figure **??**, the Kirchhoff node equation would be

$$I_1 + I_2 + I_3 = 0. \tag{2}$$

Another way of thinking of this relationship is that the current that flows into a node must be balanced by the current that flows out of the node (i.e. current does not build up or leak out of a node in a circuit). This means that at least one of the currents in the current equation will be negative and represent the current flowing out of the node. This expression is an application of the conservation of charge.

Kirchhoff's second law, which is similar to his first law, states that the sum of all voltage drops across each electrical element (such as resistors, capacitors, batteries, etc.) in a circuit loop must be zero. A mechanical equivalent to this is a roller coaster. The car in a roller coaster is initially pulled to the top of a hill. In a circuit, this is represented by the battery placing the charges at the top of a potential 'hill'. Once the roller coaster reaches the top of the hill, it begins to fall and move through various twists and turns. In the circuit, these twists and turns can be thought of as the various elements, such as resistors, within the circuit. After moving through the various twists and turns, the roller coaster car ends at the bottom of the hill where it initially started. In a similar manner, the charges in a circuit end up at the 'bottom' of the potential hill in a circuit.

In order to sum the voltages around a loop, the voltage polarity of each object must be known (i.e. we must determine if the charges are 'moving uphill' or 'moving downhill' in our roller coaster analogy as we go around the loop). For a battery, the polarity is usually indicated on the battery with a "+" or "-" near one of the terminals. On a circuit diagram, the different terminals are represented by the size of the plate. The larger plate indicates a positive terminal, while a smaller plate indicates a negative terminal. When going around a loop, the sign we end up on as we go across the battery is the polarity of the battery in the loop. Figure **??** illustrates this convention.
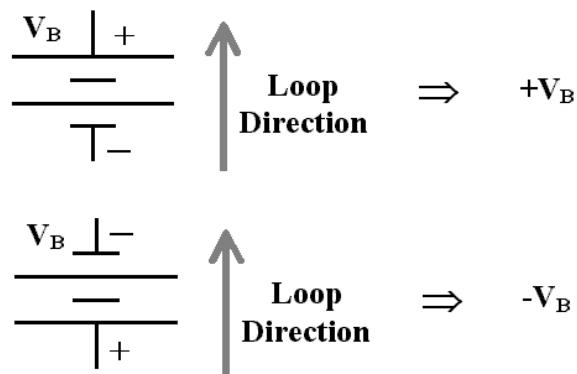
Figure 2: Battery voltage polarity convention

The direction of current flow through a resistor determines the polarity of resistors in a circuit. For these types of problems, current is thought to be the flow of positive charges. In actuality negatively charged electrons flow, but this was not known when Kirchhoff made his discovery. The discrepancy here does not effect the results, but should be kept in mind. If we consider the current to be made up of positive charges flowing through the wires, then the charges will move from higher, "+", potential to lower, "-", potential. Just as in batteries, the sign we end up on as we go around the loop will determine the polarity of the resistor. This convention is illustrated in Figure **??**.
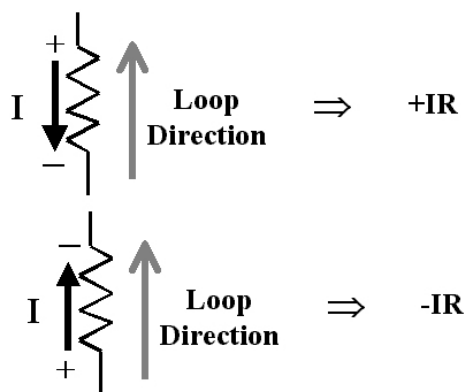

Figure 3: Resistor voltage polarity convention

The application of these polarity conventions on the circuit shown in Figure **??** are shown in Figure **??**
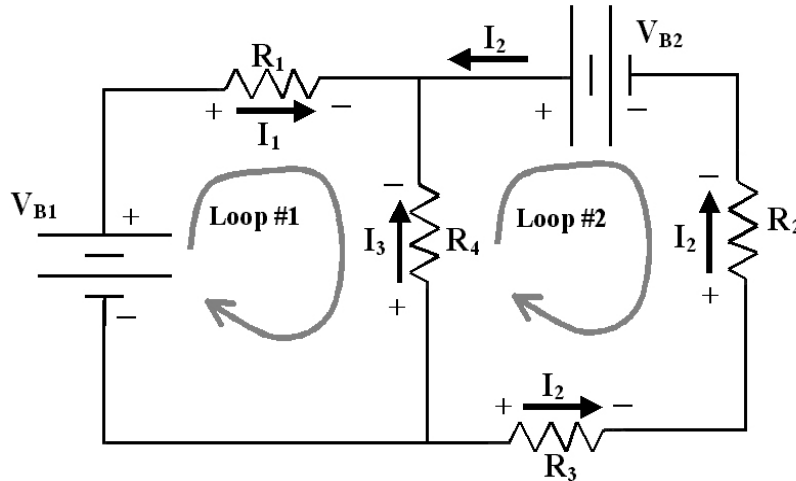

Figure 4: Example of application of voltage polarity convention

Applying Kirchhoff's voltage law to this circuit gives the following equations:
For loop # 1,

$$\sum_{i=1}^{N} V_i = 0 \implies V_{B1} - I_1 R_1 + I_3 R_4 = 0, \tag{3}$$

For loop # 2,

$$\sum_{i=1}^{N} V_i = 0 \implies -V_{B2} + I_2 R_2 + I_2 R_3 - I_3 R_4 = 0. \tag{4}$$

Combining these equations with the current/node equation we obtain the following:

$$\sum_{i=1}^{N} I_i = 0 \implies I_1 + I_2 + I_3 = 0. \tag{5}$$

Here we have three equations that describe the voltages and currents in the circuit. In a typical Kirchhoff's laws problem, the battery voltages and the values of the resistances are known, while the currents are not known. In this situation, the application of Kirchhoff's laws would give several equations that could be used to solve for the currents. One voltage/loop equation is produced for each loop in the circuit. The number of current/node equations is typically one less than the number of voltage/loop equations.

In this set of experiments, Kirchhoff voltage and current laws will be experimentally verified for a typical circuit containing two loops. Additionally, Kirchhoff's laws will be applied to solve for the currents in a number of different circuits, and these current values will be compared to experimentally measured values.

Procedure and Data Analysis

Verification of Kirchhoff's Laws

Current/Node Law

In this experiment, Kirchhoff current/node law, which states that the sum of the currents flowing into a node must be zero, will be tested experimentally.

1. Using the multimeter, measure the value of the resistance of each of the three resistors provided by setting the scale of the multimeter on the **200K** scale.

2. Use the multimeter to measure the voltage from the battery(s) in the single D battery holders and the two D battery holder.

3. Set up the circuit shown in Figure **??**. In this circuit, use one of the single D battery holder for V_{B1a} and the two D battery holder for V_{B2}. For the resistors in the circuit, use the resistors closest to the following values: $R_1 = 50 \ k\Omega$, $R_2 = 20k\Omega$, and $R_3 = 10 \ k\Omega$.
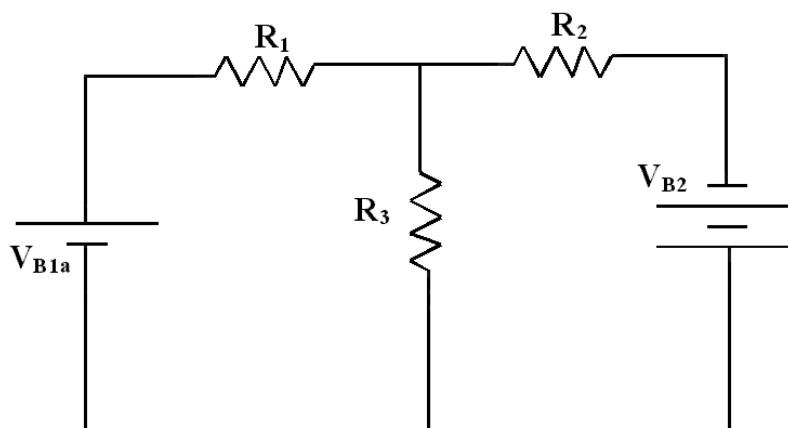

Figure 5: Circuit for verification of Kirchhoff's Laws

4. Set the multimeter on **200 u** on the current scale (i.e. 'the '**A**" scale). Attach a black lead to the **COM** terminal and a red lead to the **mA** terminal. With these settings, the multimeter is set to read the current in the circuit in micro Amperes (i.e. μA).

5. Measure the current flowing into the top node of the circuit from each of the three branch wires. To measure the current you will have to break the circuit to insert the multimeter. You must also measure the polarity of the current in a consistent manner. **If the current flows into the node, then the current should be measured from the positive (red/mA) terminal to the negative (black/COM) terminal.** Record these measured currents on your data table.

6. In the space provided on the data table, add the three currents to check that the sum of the currents is zero (or close to zero).

7. If your data results do not comply with theory confer your discrepancy with the Lab Instructor or Lab TA.

Voltage/Loop Law

1. Pull the red lead out of the **mA** terminal on the multimeter and place it in the **V Ω** terminal. Set the multimeter to the 20 volt scale.

2. Using the circuit shown in Figure **??**, measure the voltage drops across each element in each loop. The polarity of the voltage must be measured in a consistent manner. **In order to measure the voltage in a consistent manner, always place the positive lead (i.e. the red or V Ω lead) in front of the element, relative to the arrow while the negative lead (i.e. the black or COM lead) is behind the element, relative to the arrow. If this procedure causes difficulties, consult your Lab Instructor or Lab TA for assistance.** Record the voltage drop across each element, for each loop, in the space provided on the data table.

3. For each loop, add the voltage drops across each element to verify that the voltage adds to zero (or close to zero). If your data results do not comply with theory, confer your discrepancy with the Lab Instructor or Lab TA.

## Calculation of Current Using Kirchhoff's Laws

In this series of experiments, Kirchhoff's Laws will be used to calculate theoretical values of current flowing into a node. The theoretical results will be confirmed through experimentation.

1. Using the values of the voltages from the battery holders and the resistance of the resistors, apply Kirchhoff's Laws to one of the circuits shown in Figure **??** to generate the voltage and current equations for the circuit. Each person in a lab group should choose a different circuit to work with. Each person should choose only one circuit. There may be some circuits in the figure that are not used. If you have trouble generating the equations, consult with the members of your lab group or with the Lab Instructor or Lab TA.

2. Using these equations, solve for the current flowing into the top node of the circuit. If you have trouble solving for the currents, consult with the members of your lab group or with the Lab Instructor or Lab TA.

3. Record the theoretical values of the currents in the space provided in the data table.

4. Set up each circuit and measure the current through the node at the top of the circuit. In these circuits, use the single D battery holder for V_{B1a} and the two D battery holder for V_{B2}. For the resistors in the circuit, use the resistors closest to the following values: $R_1 = 50 \ k\Omega, R_2 = 20k\Omega$, and $R_3 = 10 \ k\Omega$. Record the experimental values of the currents in the space provided.

5. For each current, calculate the percent difference between the theoretical and the experimental values of the current. Record these values in the space provided.

6. Repeat this process for the circuits shown in figure **??**. Each person within a lab group should choose a circuit from the assortment of both cases and individually produce all required data for the circuits analyzed. It is not necessary to analyze every circuit. Be sure to include a sketch of the analyzed circuit in the spaces provided on the data table.

Selected Questions

1. For the following circuit, set up the appropriate number of current/node equations and voltate/loop equations on a separate sheet of paper. **Do not** attempt to evaluate the current values.
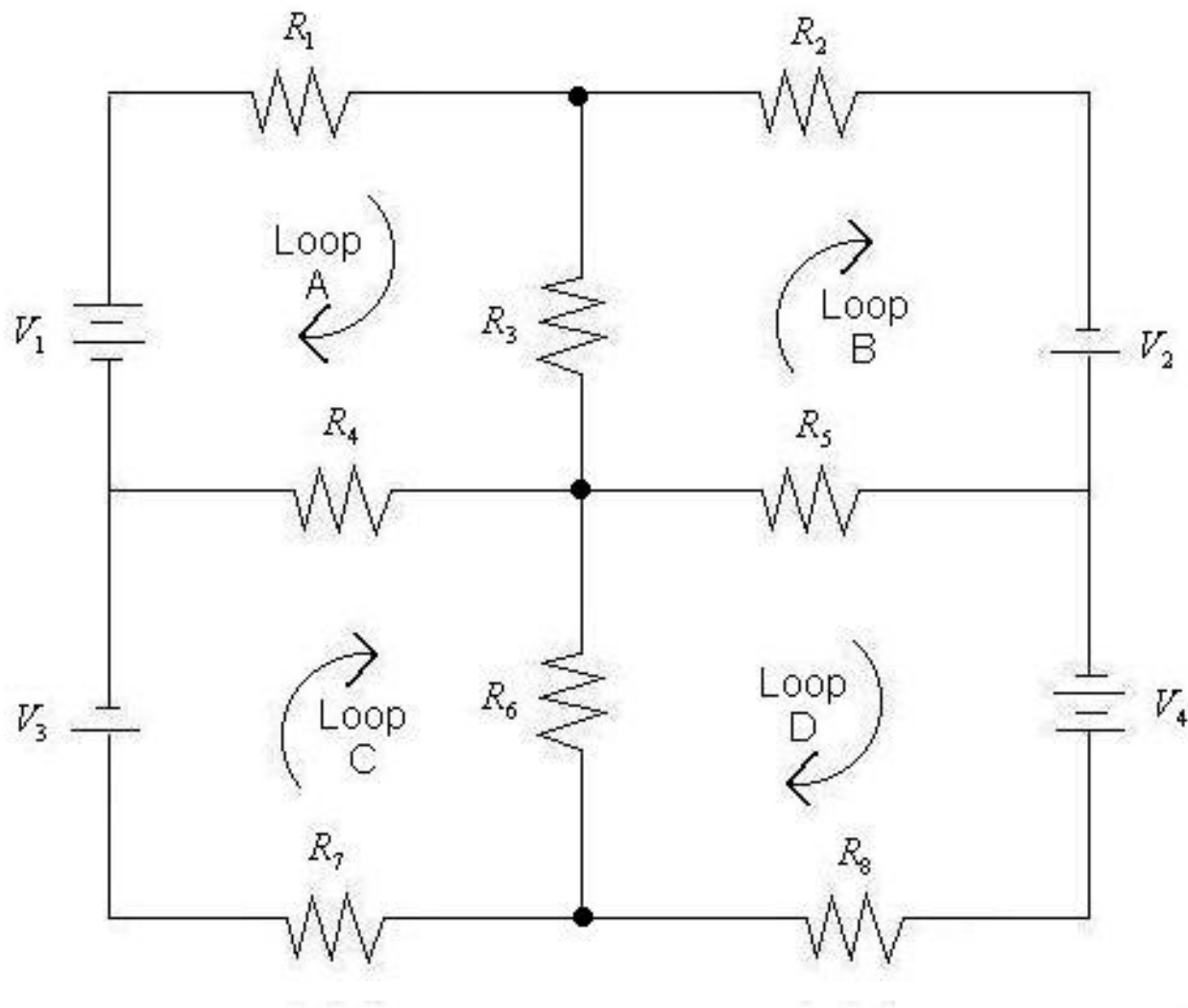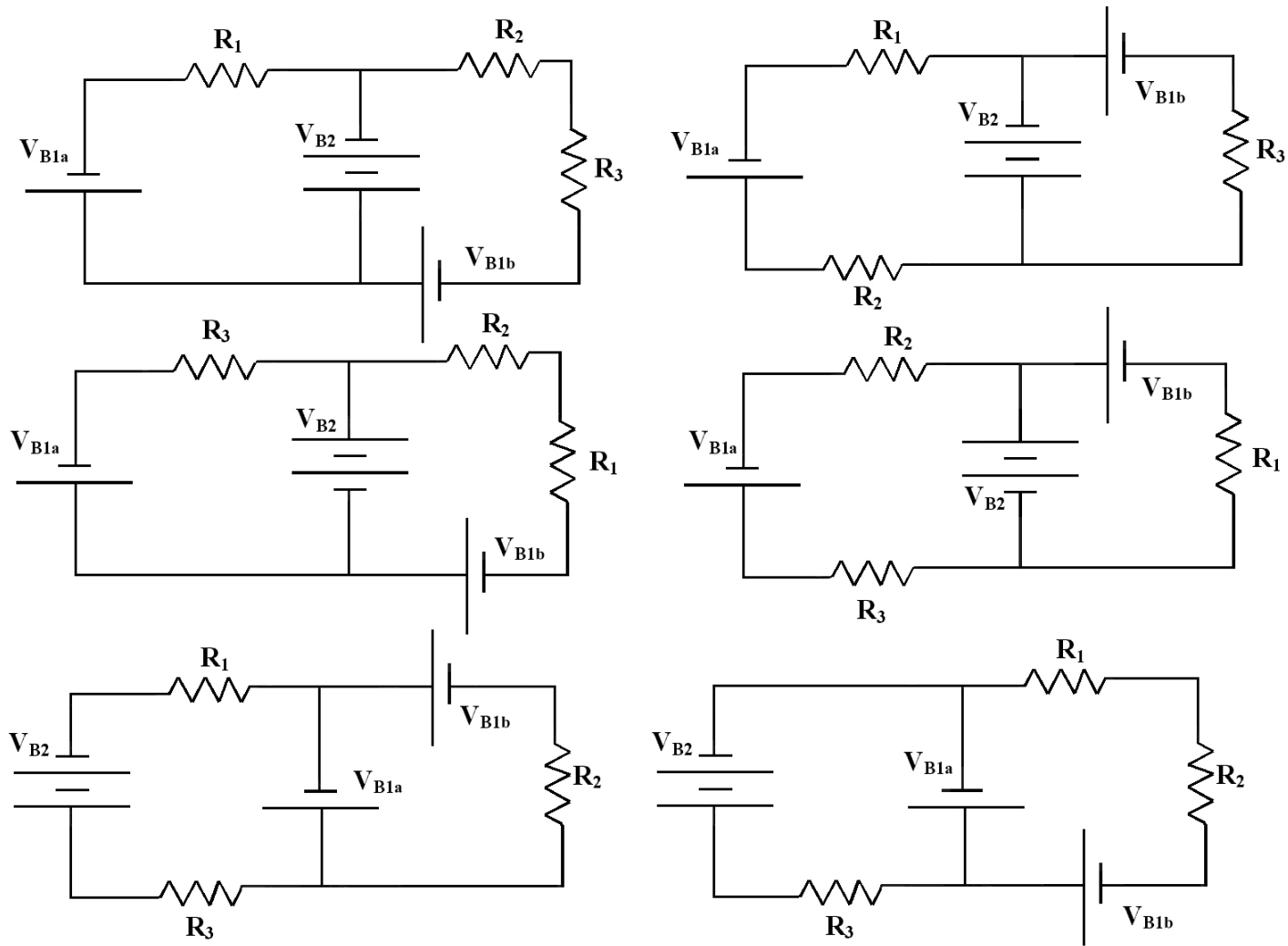
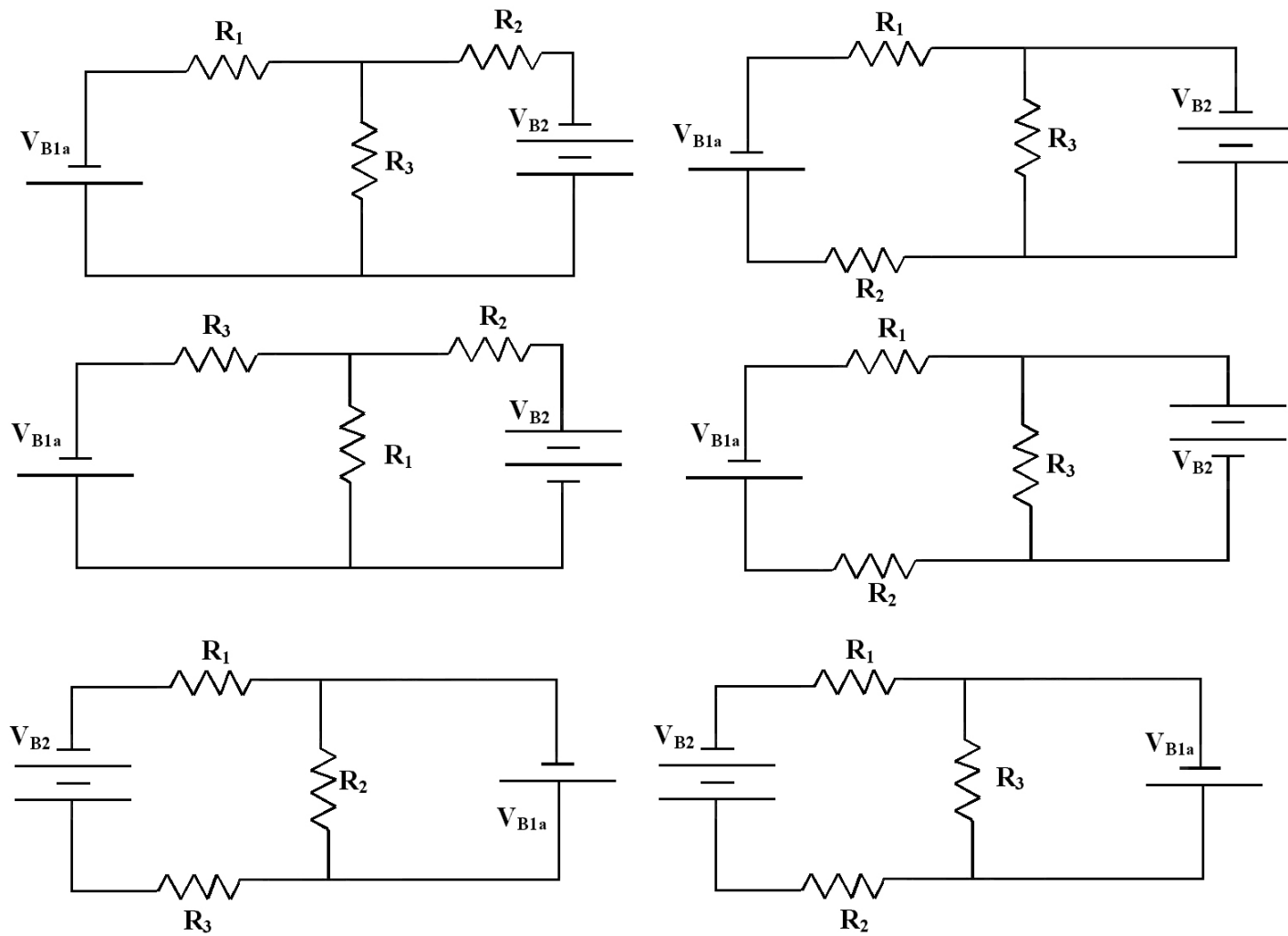Figure 6: Circuits for current prediction from Kirchhoff's Laws: Case 1

Figure 7: Circuits for current prediction from Kirchhoff's Laws: Case 2