Faraday's Law and Joule Heating

1 Objective

Using careful measurements allowed only by digital technology, this lab will consider voltage pulses generated by a time changing magnetic flux. Pulse characteristics will be studied in relationship to Faraday's law, and Joule heating will also be studied when currents induced by the pulses are dissipated in a resistive load. Finally, because Faraday's Law involves magnetism, an enquiry feature of the lab is included. It investigates the difference in character of the induced voltage pulse according to the influence of a bar's material type, whether brass or steel.

2 Equipment List

A multi-turn coil of wire; a rare-earth magnet attached to the end of either (i) a brass or (ii) steel bar; that is inserted into one end of a wooden-bar; whose other end contains a ball-bearing; a smart timer with photogate; a digital oscilloscope with communications module that allows data to be transferred to the computer; a load resistor (nominally 25 ohms); a multimeter

2.1 Description

A magnet is caused to pass in simple harmonic motion through a multi-turn stationary coil of wire. The magnet is atached to a pendulum whose motion in free-decay is studied with either (i) a smart timer or (ii) a digital oscilloscope.

2.2 Smart timer velocity measurements

The decline of maximum velocity of the pendulum is determined by means of the smart timer operating in the `stopwatch' mode. The photogate is placed at the position of the lower end of the wooden bar corresponding to static equilibrium of the pendulum.

When the pendulum is swinging, the `trimmed-down' small-end of the wooden bar occults the infrared beam of the photogate, after which the timer displays an interrupt time Δt. It is assumed that the speed during interrupt is inversely proportional to Δt.

Values of Δt are recorded manually, for every passage-in-the -same-direction through the photogate if you are nimble, or every 2nd passage otherwise. The recording of these numbers requires two team members-one to control the smart timer and verbally announce each reading, and the other to write-down on a piece of paper each value of Δt as s/he hears the number.

3 Theory

3.1 Faraday's Law

The law you will study is expressed as

$$\text{`emf' voltage} = -N \frac{d\Phi_B}{dt} \qquad (1)$$

where Φ_B is the magnetic flux associated with the multi-turn coil in which a voltage is induced (referred to historically as the `electromotive force' or `emf ', not really a force). The flux is defined by the following area integral

$$\Phi_B = \int \mathbf{B} \cdot d\mathbf{A} \qquad (2)$$

The negative sign in Eq.(1) expresses the consequence of energy conservation and represents Lenz's law. Lenz's law will be evident in this experiment as one observes the extra damping of the pendulum that results when the coil is shunted by a load resistor.

3.2 Oscilloscope velocity measurements

As seen from Eq.(1), the induced voltage is in magnitude equal to the time derivative of the magnetic flux. For this experiment the area is constant, and the magnetic field at a fixed distance from the magnet (in a given direction) is invariant. Therefore the change in flux derives from the spatial dependence of the field, and thus the voltage induced by the magnet should be nearly proportional to its velocity. For this reason it is assumed that by measuring the voltage, the oscilloscope also is measuring the velocity. Only the relative change is required for several parts of this lab, and by measuring the decline of the peak voltages during free decay, a test of Faraday's law will be performed - by comparing this decline to the decline determined with the smart timer.

3.3 Pendulum energy

Friction in the bearing causes the pendulum to lose energy during free-decay. The rate of energy loss increases even more than what derives from this mechanical source, due to Lenz's law, when current is dissipated in the form of heat, by shunting the coil with a load resistor. You will measure the rate of decline with and without resistance, as one of several means for testing Faraday's law.

The total energy of the pendulum, which interchanges periodically between potential and kinetic forms, can be expressed as follows:

$$E(t) \; = \; \frac{1}{2} \, I \, \omega^2 \, \theta_0(t)^2 \qquad\qquad (3)$$

where θ_0 is the amplitude (maximum angular displacement in radians), $\omega \; = \; 2\,\pi\,/\,T$, and I is the moment of inertia. For the relatively small amplitudes considered in this study, the period T may be assumed constant as an acceptable approximation. Because measurements are made in free-decay as a function of time, θ_0 and therefore E are time dependent. You will determine by measurement the nature of the turning-points of the motion, and in so doing will demonstrate that not all decays are exponential, as is commonly believed.

The free-decay will be studied for three different conditions, the first case being one in which (i) the coil is effectively `open circuit' during data-collection; the second case being one that has (ii) a load resistor of nominally 25 Ω connected across the coil; and for the third case (iii) the coil is short-circuited.

3.4 Example Smart-timer results

Shown in Fig. 1 are example results using the smart timer, in which each of the velocity graphs has been normalized. The units are inconsequential since only the relative variation in time is needed.
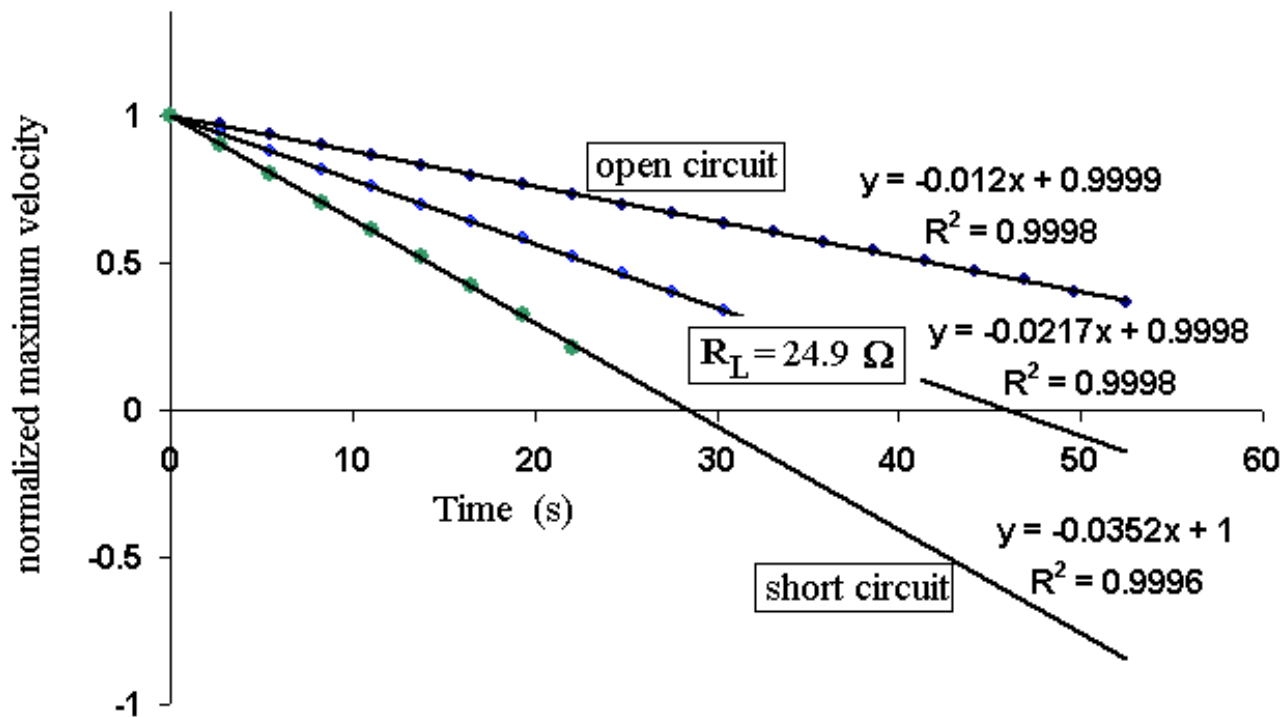
Figure 1. Time variation of the maximum speed of the pendulum for the three study-cases. These data were generated with the smart timer.

For the study that produced the example results, the period of the pendulum was measured (also with the smart timer in the pendulum mode) to be T = 1.38 s. Using a meter stick to measure the displacement of the magnet away from equilibrium, the initial amplitude was estimated to be $\theta_0(0)$ = 0.18 rad. From the values for the trend-line slopes shown in Fig. 1, one thus obtains the following first-cycle decrements

$$\Delta\theta_{0,1} = -0.0120 \times 1.38 \times 0.18 = -0.00298 , \quad \Delta\theta_{0,2} = -0.0217 \times 1.38 \times 0.18 = -0.00539 ,$$
$$\Delta\theta_{0,3} = 0.0352 \times 1.38 \times 0.18 = -0.00874$$

(4)

4 Test of Assumptions

To validate the assumptions on which this experiment is based, it is necessary to show that the decline of voltage-pulse maxima (as measured with the oscilloscope) occurs at the same rate (within error) as the decline in maximum speed of the magnet (as measured with the smart timer). Shown in Fig. 2 is an example of such a test.
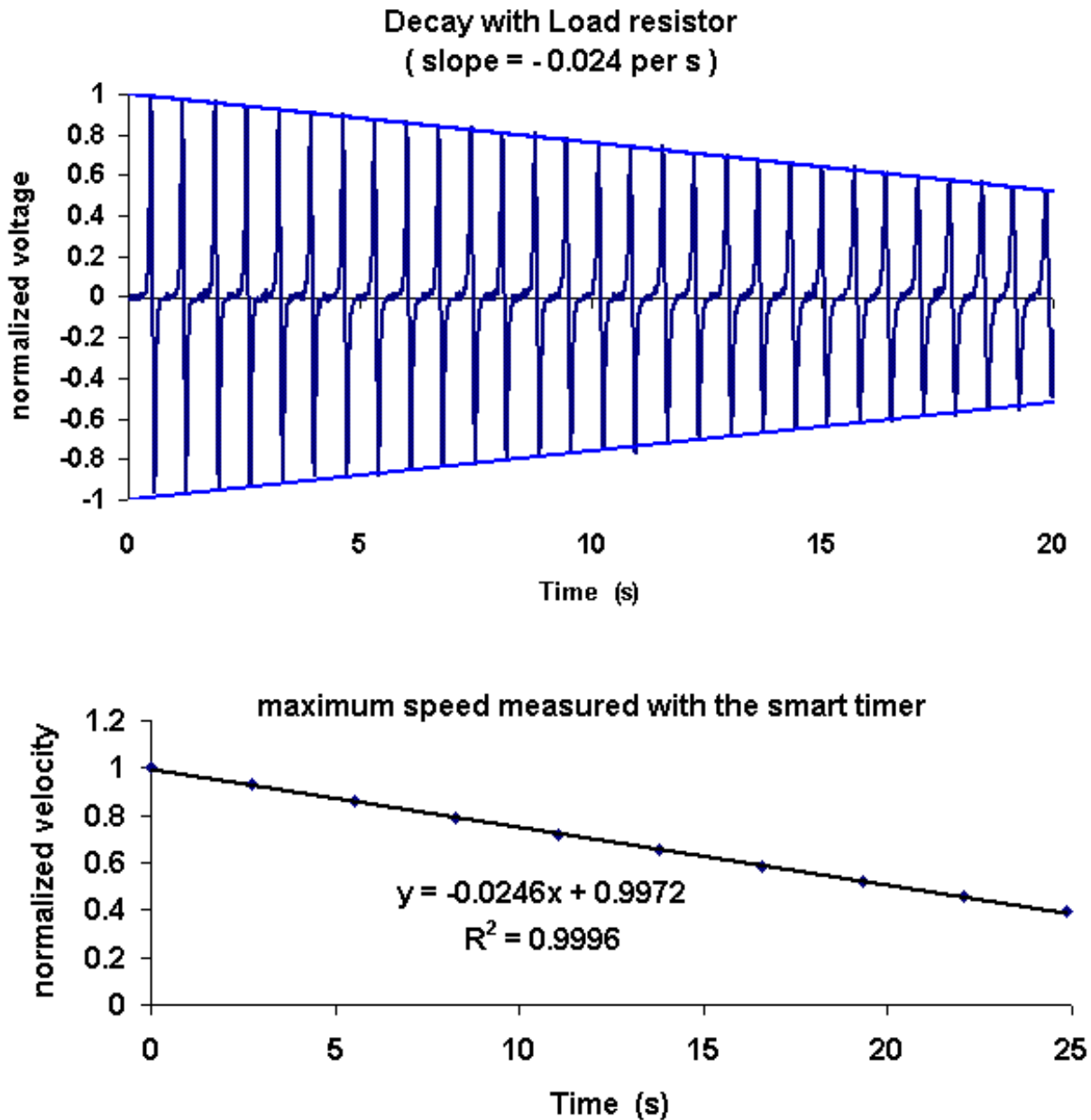
Decay with Load resistor
(slope = - 0.024 per s)

maximum speed measured with the smart timer

$$y = -0.0246x + 0.9972$$
$$R^2 = 0.9996$$

Figure 2. Comparison of a voltage decay from the oscilloscope (top graph) against a maximum speed decay from the smart timer (bottom graph).

The graphs were all generated with Excel, with the oscilloscope data being transferred to the computer using the flash card. The straight blue lines corresponding to the turning points of the voltage were produced by generating linear functions with slopes (equal in absolute value) that were adjusted by trial and error. The `best' (`eyeball' fit) is strictly a visual result.

4.1 Electrical noise

Fig. 2 shows only a decay for which the load resistor was in place. In the open circuit case, the coil behaves like an antenna and so there is an order of magnitude greater noise on the oscilloscope as illustrated in Fig. 3.
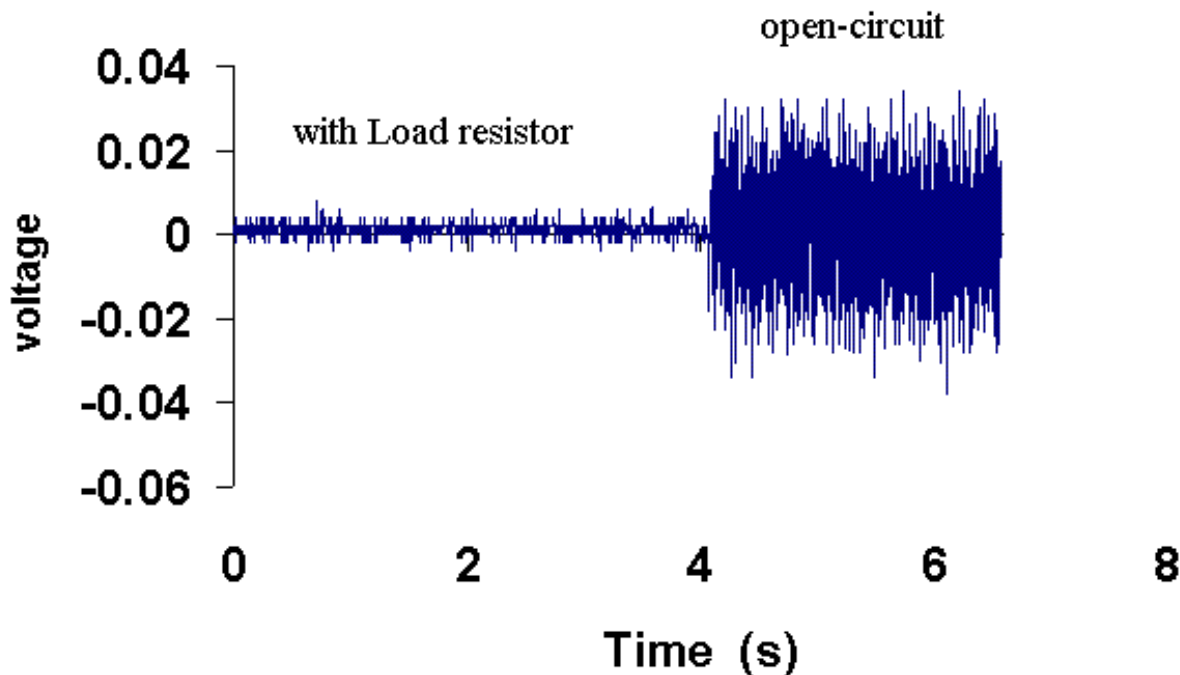
Figure 3. Showing how the noise is much greater for the coil open-circuit as opposed to the case in which it is shunted by a 25 Ω load resistor.

The noise of the open-circuit case makes for a difficult measurement of the voltage decline and so it is not used. And of course there is no voltage applied to the oscilloscope when the coil is shorted. Thus only a load resistor case will be considered as you assess the acceptability of using timer data to validate Faraday's law.

5 Experimental Procedure

Velocity decay measurements using the Smart Timer

(1) With the pendulum resting in equilibrium, check the location of the photogate to insure that its infrared beam is close to the point of occultation if the pendulum were swinging.

(2) Manually displace the pendulum to your chosen starting position that establishes the initial displacement amplitude $\theta_0(0)$ for all data runs. For this purpose it is convenient to use a (measureable) starting displacement of the magnet from the end of the coil form. This displacement should correspond roughly to pendulum displacement $\theta_0(0) = 0.2$ rad. An easy way to initialize the pendulum to this amplitude every run is to place a thin strip of tape on the brass or steel rod and let this strip line up with the end of the coil form as you release the pendulum from rest.

(3) Release the pendulum and make any necessary adjustments to insure that the magnet passes through the coil without striking anything as the pendulum swings.

(4) Having established that your configuration is operational, set the pendulum into motion and use the smart timer to **measure the period**. For this mode the timer is set to (i) time, (ii) pendulum. Do this a few times (or enough) to establish an estimate for the period that you believe to be good to three significant figures. Record your mean value with an estimate of its uncertainty.

(5) With the smart timer, collect the data (Δt values) that establish the decay characteristics of the system. The first run will be **without** the Load resistor, the second run will be **with** the load resistor shunting the coil, and the third run will be with the coil **shorted**. The mode of the timer for these runs will be (i) time, (ii) stopwatch.

Manually initialize the pendulum and release it to swing freely, having selected from the two possibilities, a single direction of passage through the photogate with which to operate. Before passage, the start/stop switch is depressed to clear the display and arm the timer. Following passage of the pendulum, the time duration Δt of beam occultation is displayed. This value remains displayed until the start/stop switch is again depressed.

The operator of the timer will note and communicate the Δt value to the individual recording the numbers. For less nimble operators the periodicity of the recording process is conveniently equal to twice the period of the pendulum. For example, if the pendulum's period should be 1.38 s, then a Δt number would be written manually on paper every $\delta t = 2.76$ s, on average.

Note: It is the responsibility of the timer operator to insure correlated activity between pendulum and personnel. Synchronization should be sustained for no less than one-half a minute, and for most a practice session is in order. Failure in the process will result in an Excel graph having discontinuities; if this should happen, the procedure must be repeated from the start.

(6) Type each of the sequentially recorded Δt values for the three different cases mentioned above into columns E, F, and G respectively of an Excel spreadsheet. For the A column, starting with a zero in A1, generate the time sequence associated with the numbers placed in the E through G columns. To generate the A-column numbers, go to A2 and type `` = A1+δt'' using for δt the number appropriate to your system. Then fill the column as follows. With the mouse go to the lower right corner of the occupied A2 and when the cross of the cursor changes shape-holding the left mouse button, drag downward and autofill to the maximum time to be considered.

Place in H1 ``n.factor1''. In H2 place the number 1 as a starting `guess'. Do the same for rows 1 and 2 of columns I and J, with I1 being ``n.factor2'' and J1 being ``n.factor3''. In B1 type " = 1/E1/$H$2 ". Then fill the B column. This can be done as with the A column using `autofill'. An easier way to fill, once A is already fully occupied- is to place the cursor at the lower right corner of B1 and double click the left mouse button. If any `DIV/0' rows should result, simply highlight and delete them. Repeat for the C column after typing into C1 " = 1/F1/$I$2 ". Finally, repeat the process for the D column after typing into D1 " = 1/G1/$J$2". Now generate a graph showing points-only using columns A, B, C, and D. Do this by clicking on A, and with the ctrl button depressed, clicking also on B, C, and D. Looking at the starting values of the graph, change by trial and error the numbers in H2, I2, and J2 until the three nearly-straight set of points begin at unity. This is easily done by watching the monitor when you hit `enter' after changing a given number.

Once you are satisfied with your trial and error selections, do a trendline on each of the three data sets. Your final result should look similar to Fig. 1 above.

For your report, generate a graph showing the three decays that correspond to your system.

Voltage decay measurements using the Oscilloscope

With a nominal 25 Ω resistor across the coil, initialize your pendulum using the brass rod and generate a waveform on the oscilloscope similar to the one pictured in Fig. 4. For data-capture, you will need to hit the run/stop button on the instrument panel.
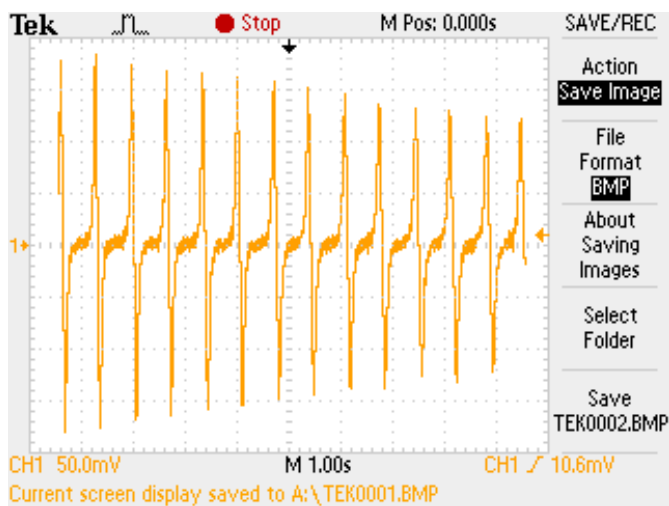

Figure 4. Example waveform, using the brass rod. Convenient settings of the oscilloscope are 50 mV/div and 1 s/div.

For the figure supplied with this document the `save' mode used was one of `image'. In your case save the `waveform' instead and transfer it to the computer using the flash card. Once in the computer open the file with Excel and generate a graph like the top case of Fig. 2.

Estimate the decay of the turning points by calculating the slope of your fit and compare this slope to the trendline value you obtained for the same decay using the smart timer. You may estimate your slope by whatever means you choose. One option is to produce two straight lines (as in Fig. 2) using numbers generated and placed in columns adjacent to the voltage column. Then all three are graphed together with the best trial and error fit as was used previously in analyzing the smart timer data. Another option is to lay a ruler on the display showing the turning points (monitor or printed copy) and estimate the decline by reading off numbers from the scales of the display.

For your report, as measured for your pendulum, generate two graphs like the ones shown in Fig. 2.; be sure to state whether or not you think the two graphs yield self-consistent results, relative to the goals of this experiment.

Data using the Steel rod

Using a sweep speed for the oscilloscope of 250 ms/div, capture the first pulse after letting go of the pendulum. Do this for both the brass rod and the steel rod. With Excel plot the voltage versus time for your emf pulses, which should look similar to the ones shown in Fig.'s 5 and 6.
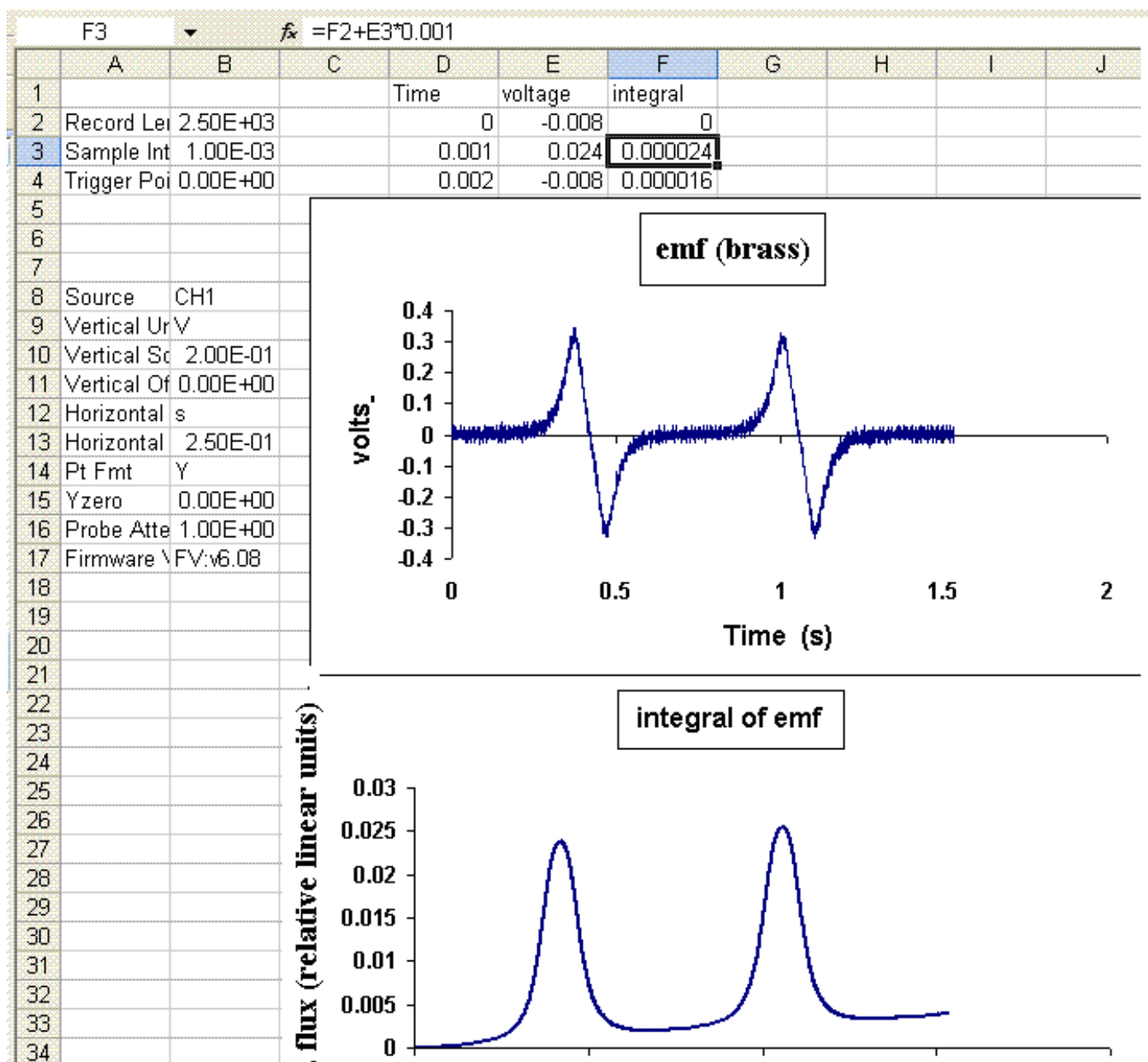
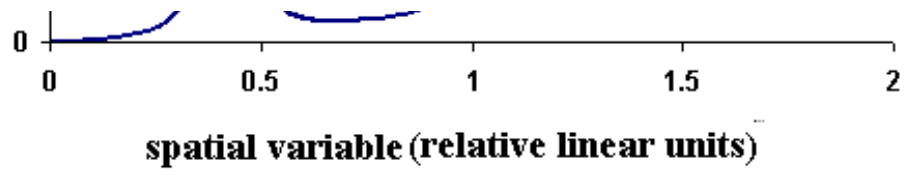|	
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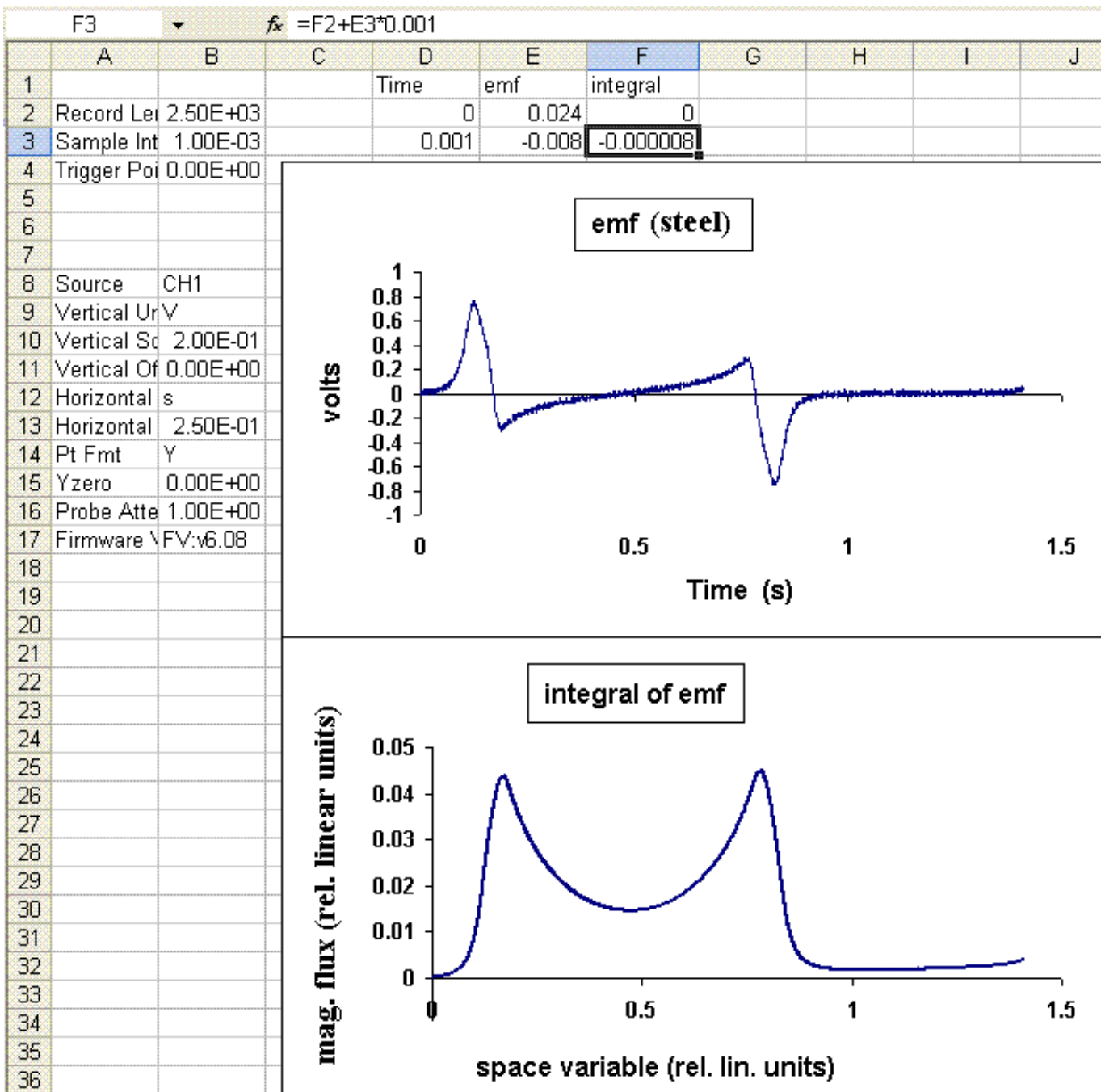**Figure 5**. The emf and its integral using the brass rod.

Figure 6. The emf and its integral using the steel rod.

For each of the cases, also do an `integral' over the emf in the manner implied by the entry at F2; i.e., `` = F2+E3*.001" (assuming that the time increment for your data is 1 ms.) This integral is a reasonable approximation of the flux surrounding the magnet (when the independent time axis is transformed to a space variable.)

For your report you will print all four of these graphs.

QUESTION: In keeping with the enquiry exercises of previous labs- (1) Did you expect the difference seen between Figures 5 and 6. (2) How do you explain the differences?

Appendix
Quantitative comparison of Theory and Experiment
For extra credit as determined by your instructor, you may wish to do the quantitative comparison described here.

The rate at which pendulum energy decreases is different for the cases of with and without a load resistor. The difference is governed by the following equations, if $\Delta\theta_0$ is as assumed, small compared to θ_0:

$$\Delta E_1 = I \omega^2 \theta_{0,1} \Delta\theta_{0,1} \quad , \quad \text{without load resistor} \qquad (5)$$

$$\Delta E_2 = I \omega^2 \theta_{0,2} \Delta\theta_{0,2} \quad , \quad \text{with load resistor} \qquad (6)$$

$$\Delta E_2 = \Delta E_1 + \Delta E_R \qquad (7)$$

Here ΔE_R is the energy dissipated per cycle in all of (i) the load resistor R_L, (ii) the coil resistance R_c, and (iii) the resistance R_w of the hook-up wires. For the goals of this experiment, the value of R_w should be small enough to ignore.

An estimate for ΔE_R is obtained by considering the `emf' v(t), induced in accord with Faraday's Law, across the pair of resistances as shown in Fig. A1.
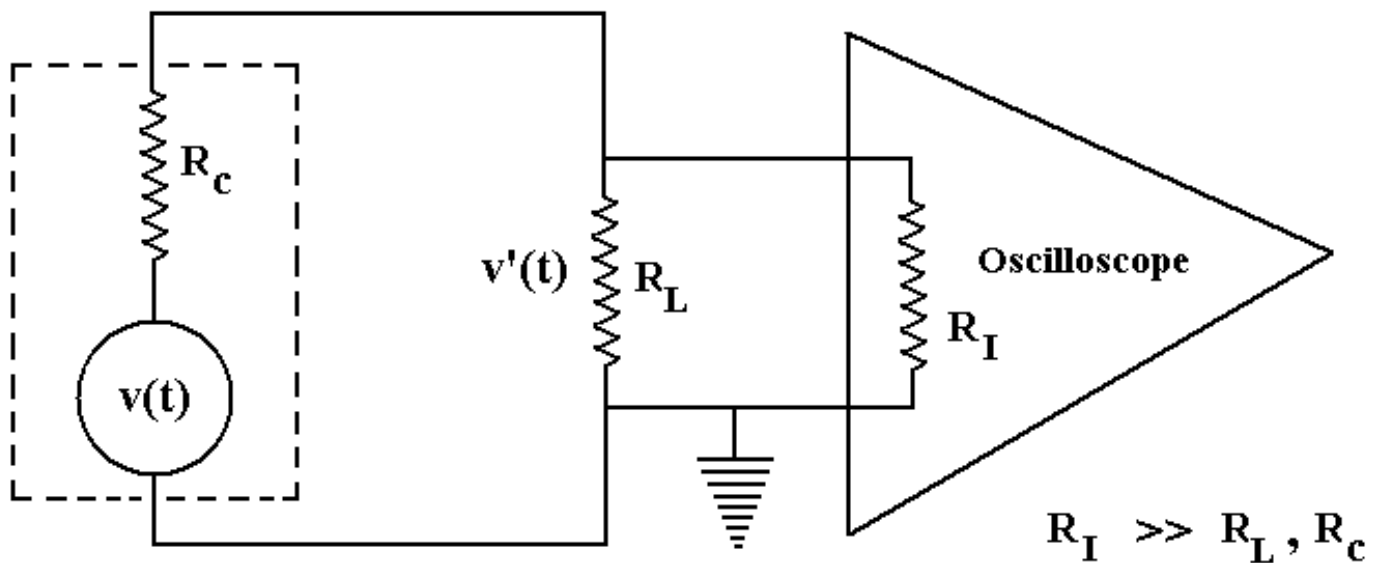

Figure A1. Electrical equivalent circuit for the system, assuming hookup wires contribute insignificant resistance.

Note that the input resistance of the oscilloscope is much greater than either of the other resistances shown. Therefore, the voltage v′(t) measured by the instrument is smaller than v(t) according to

$$v'(t) = \frac{R_L}{R_c + R_L} \, v(t) \tag{8}$$

The power dissipated as Joule heat in the two resistors is given by

$$P(t) = i(t)^2 (R_c + R_L) = v(t)^2/(R_c + R_L) \tag{9}$$

Combining eqns (7) and (8) yields

$$P(t) = \frac{v'(t)^2}{R_L} \left(1 + \frac{R_c}{R_L} \right) \tag{10}$$

After use of the preceding equations, the energy lost to heat is obtained by integrating eq.(7) over the appropriate time interval (specified later); i.e.,

$$\Delta E_R = \frac{- \left(1 + \dfrac{R_c}{R_L} \right)}{R_L} \int v'(t)^2 \, dt \tag{11}$$

To employ Eq.(11), the pendulum will be initialized to the same amplitude for every case, and the time duration over which a numerical integration will be performed is one-half of the **first decay cycle**, working with the **brass** rod and **not** the steel rod. The estimates of $\Delta\theta_{0,1}$ and $\Delta\theta_{0,2}$ will be obtained by first measuring the magnitude of the maximum velocity of the pendulum as a function of time, using the smart timer. Later a similar determination of the maximum velocity will be obtained from measurements with the oscilloscope. It should be noted that velocity estimates using the oscilloscope are not possible for the short-circuit case.

Shown in Fig. A2 (left graph, negative/positive pulse) is an Excel-generated graph of the oscilloscope-measured voltage $v'(t)$ obtained from the first half-cycle of decay with the load resistor $R_L = 24.9 \ \Omega$ connected across the coil whose resistance was $R_c = 25.8 \ \Omega$. Measurements next described are done only with the load resistor in place, since there is inconsequential heat-loss for the open-circuit case. Pendulum decay in the open circuit case is due to mechanical friction alone. The increase in the decrement when the load resistor is in place (as is obvious from Fig. 1) is due to Joule heating in R_c and R_L.

Also shown in the left graph of Fig. A2 is the square of the voltage. This double-humped, positive-only waveform was numerically integrated with Excel to yield the right-side graph of the figure.
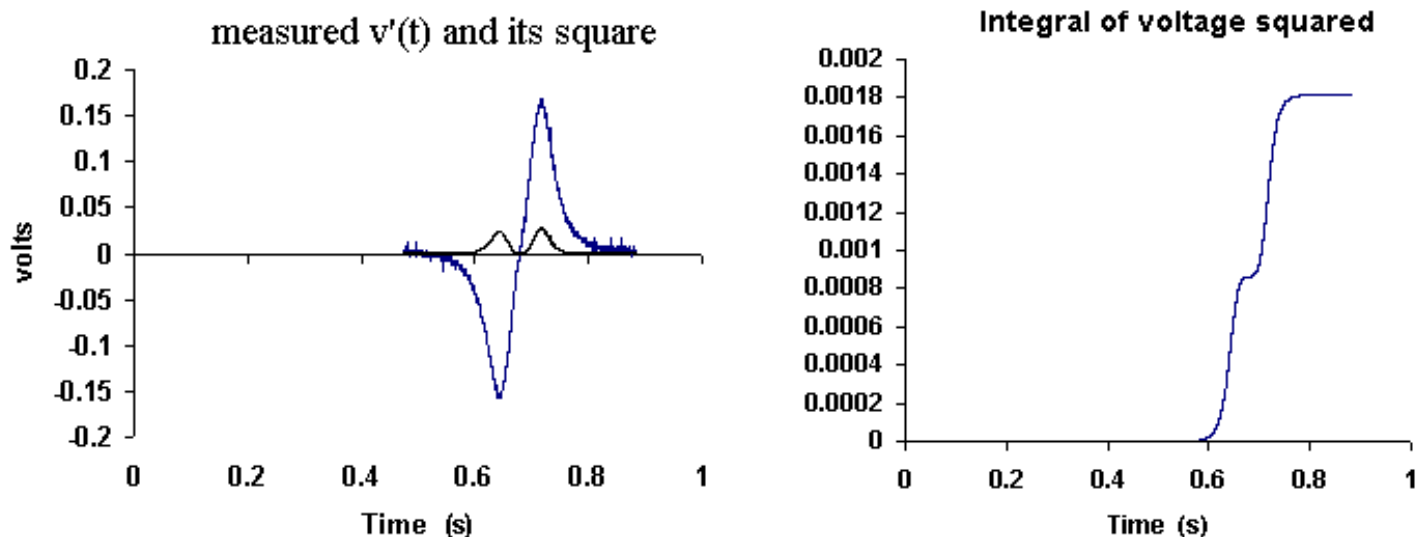
Figure A2. Example waveforms plotted with Excel following transfer of data from the oscilloscope to the computer.

`Capture' of the pulse was accomplished with an oscilloscope sweep speed of 0.1 s per division and a voltage setting of 0.1 volt per division. A significantly slower sweep speed would have resulted in too few digital data points to yield a decent numerical approximation to the integral. Similar loss of resolution would result if the input range setting had been too large such as at 1 volt per division.

The units of the integral are Joules times ohms, and the largest (final) value of 0.00182 is used to compare theory with experiment.

6 Theory vs Experiment

6.1 Free-decay method for estimating the Moment of Inertia

You will generate an estimate for the moment of inertia of your pendulum based on the rate of energy loss due to Joule heating. From Eqns (5) - (7) with $\theta_{0,1} = \theta_{0,2} = \theta_0$ and using Eq. (11), one obtains the following expression as a means for estimating the moment of inertia:

$$I = \frac{|\Delta E_R|}{\omega^2 \, \theta_0 \, (\, |\Delta\theta_{0,2}| - |\Delta\theta_{0,1}| \,)} \quad , \qquad |\Delta E_R| = \frac{(\, 1 + \dfrac{R_c}{R_L} \,)}{R_L} \int v'(t)^2 \, dt \qquad (12)$$

The range of the integral is chosen to encompass the entire features of a single pulse. It is important to exclude any non-zero parts (except for noise), both before the start of the pulse and following the end of the pulse. For the following example-case numbers (remembering to multiply by a factor of **two**, since there are two pulses per cycle):

$\omega = 4.55 \text{ s}^{-1}$, $\theta_0 = 0.180$, $|\Delta\theta_{0,2}| = 0.00539$, $|\Delta\theta_{0,1}| = 0.00298$, $R_L = 24.9 \, \Omega$, $R_c = 26.1 \, \Omega$ and for the integral over $v'(t)^2$ the value 0.00182 - we obtain for an experimental estimate of the moment of inertia

$$I_{\text{experiment}} = 0.033 \text{ kg m}^2 \qquad (13)$$

6.2 Direct calculation of the moment of inertia

To estimate I by this means requires that the pendulum be disassembled; you will not use this method

It is possible to calculate the moment of inertia of your pendulum as follows. Assume that all the mass of the brass-bar-plus-magnet M_{bar} is at the same distance from the axis r_{bar}. Thus its moment is given by

$$I_{bar} = M_{bar}\, r_{bar}{}^2 \qquad (14)$$

Since the bearing mass is distributed so close to the axis, and its mass is small - assume for the bearing that its moment of inertia is ignorable. Assume for the wooden part that it is a uniform rod of mass M_{rod} and length L rotating about one end, so that

$$I_{rod} = \frac{1}{3} M_{rod}\, L^2 \qquad (15)$$

The moment of the pendulum is then simply the sum of I_{rod} and I_{bar}.

For the example study, the indicated masses and lengths were measured to be

$$M_{rod} = 0.133 \text{ kg}, \quad L = 0.55 \text{ m}, \quad M_{bar} = 0.0656 \text{ kg}, \quad r_{bar} = 0.55 \text{ m} \qquad (16)$$

which with Eqns(14) and (15) yield the following `theoretical' estimate for the moment:

$$I_{theory,\,1} = 0.033 \text{ kg m}^2 \qquad (17)$$

6.3 Estimate of the moment based on the Period

A second theoretical estimate for the moment of inertia is obtained using the period of the pendulum. The equation of motion provides the relationship

$$I = \frac{M\,g\,l}{4\,\pi^2}\, T^2 \qquad (18)$$

where $M = 0.199$ kg is the total mass and l is the distance from the axis to the center of mass. The distance to the center of mass can be measured as follows. Disconnect the pendulum from its support and locate the balance point on the wooden rod when it is horizontal and the brass bar is hanging below it. Mark this position and measure its distance L_v from the center of the bearing. Attach the pendulum again to its support and let it hang in equilibrium. Then measure the horizontal distance L_h from the center of the wood at the balance point to the center of the vertical upright steel bar that supports the pendulum. The distance from the axis to the center of mass of the pendulum is then given by $l = (L_v^2 + L_h^2)^{1/2}$.

For the example case above, these numbers were $L_v = 34.8$ cm and $L_h = 6.5$ cm. Substitution into Eq.(18), using for the total mass M = 0.199 kg and T = 1.38 s gives our second theoretical estimate for the pendulum as

$$I_{theory,2} = 0.033 \text{ kg m}^2 \qquad (19)$$

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On 16 Nov 2007, 16:03.