

DEVICE Newton's Cradle (The Executive Toy)

TOPIC Conservation of Energy and Momentum, Mechanics

THEORETICAL BACKGROUND I wish to present two conservation principles as guides for understanding the physics of the Newton's Cradle. First, the conservation of linear momentum states that in the absence of external forces which interfere with the action of a system of particles, the total linear momentum \vec{P} of the system remains constant. Second, if the total kinetic energy of the system of two colliding bodies is unchanged by the collision then the kinetic energy of the system is conserved. In elastic collisions of several objects, kinetic energy is conserved whereas energy is not conserved in inelastic collisions. Consider a simple system of two objects of equal mass and material, namely m_1 and m_2 . For this system the conservation of linear momentum becomes

$$\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f} \quad (1)$$

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}. \quad (2)$$

Suppose further that we have a system where m_2 is at rest with an initial velocity of 0, and we wish to fire m_1 directly into m_2 creating a collision between the two. Now we say,

$$m_1 \vec{v}_{1i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \quad (3)$$

such that v_{1i} is the initial velocity of m_1 , v_{1f} is the final velocity of m_1 , and v_{2f} is the final velocity of m_2 . This equation states that the net linear momentum of the system is conserved. Furthermore, an equation stating conservation of kinetic energy for elastic collisions can be written as:

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad (4)$$

Let us now further our development by examining the special case where m_1 equals m_2 . First we must re-write equations (1) and (2) so as to solve for v_{1f} and v_{2f} in terms of the mass and the initial velocity:

$$\vec{v}_{1f} = \frac{m_1 - m_2}{m_1 + m_2} \vec{v}_{1i} \quad (5)$$

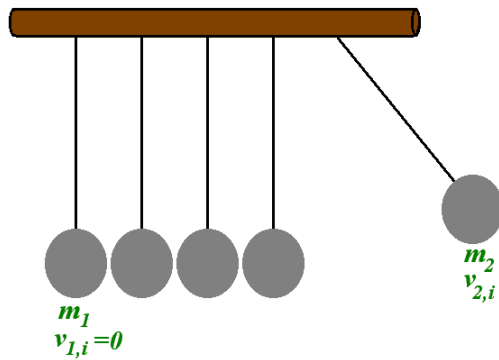
$$\vec{v}_{2f} = \frac{2m}{m_1 + m_2} \vec{v}_{1i} \quad (6)$$

Now, if m_1 equals m_2 then from equations 3 and 4 we can deduce that

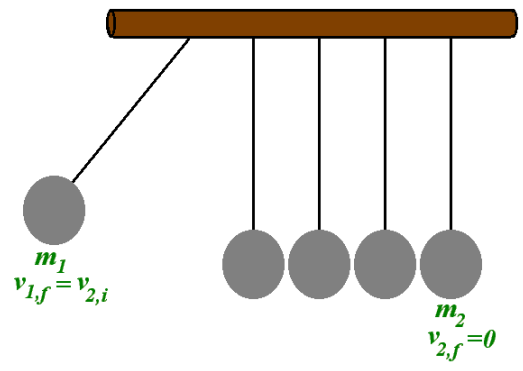
$$\vec{v}_{1f} = 0 \quad (7)$$

$$\vec{v}_{2f} = \vec{v}_{1i} \quad (8)$$

This equation is the result of a head-on collision of objects with equal masses. Here m_1 which carries the initial velocity stops dead in its tracks and m_2 which is initially at rest takes off with the speed of m_1 , thus the objects merely exchange velocities. Newton's Cradle illustrates the phenomenon of momentum conservation using a system of five balls of equal mass.



Conservation of Momentum
 $m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}$
 if $m_1 = m_2$ then $v_{1,f} = v_{2,i}$, $v_{2,f} = 0$



DESCRIPTION Newton's Cradle is a simple, yet extraordinary demonstration of the conservation of energy and momentum. Five steel balls are made to swing with fishing wire from a wooden frame post. After some time internal friction overwhelms the practical dynamics of investigation. External forces namely the force of gravity enter the system and momentum is no longer conserved, thus elastic collisions between neighbors cease.

PROCEDURE

1. After obtaining your very own Cradle, you should need no instruction as to how to operate it unless of course you are a theorist. Begin by releasing one ball from the left and observe what happens on the right. The initial energy of the system is increased by releasing the ball with a greater potential.

SUGGESTIONS Try releasing two or three balls at once from the left....