Quantum Theory of Observers

Jose L. Balduz Jr
Department of Physics
Mercer University, Macon, GA 31207, USA

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Abstract

This is an attempt to create a consistent and non-trivial extension of quantum theory, describing in detail the quantum measurement process. A tentative but concrete model is presented, based on the concept of multiple observer/participators, represented by separate state vectors. The evolution is deterministic, and in the chaotic regime implies approximate adherence to the Born rule for probabilities. The model is applied in a number of contexts: simple detectors, multi-state selectors, and intermittent systems. The results are consistent with phenomenology. We also consider more speculative applications, including specific spin and position 'observables.' Finally the outlook for the model is discussed, and its relation to other work.

The motivation for this work is the lack of a unified dynamics in standard quantum theory, a fixed rule specifying the change of the system with time. Instead there are two incompatible rules: the Schroedinger Equation(SE) whenever no measurement is taking place, and the Born rule, which prescribes that the result of a measurement is random but occurs with frequencies given by the Born probabilities(BP). In the search for a unified dynamics we assume SE must be modified to yield measurement results. Thus we define the *multiple observer model*(MOM), in the Introduction(§§1, 2) and Part I(§§3-7): No attempt is made to describe the full history of any particular measurement, rather the collapse process itself is described by use of MOM. In Part II(§§8-9), MOM is applied to more realistic measurement processes. In the Conclusion(§§10-13) the MOM is placed in context. General principles and specific competing points of view are covered, and then a classification scheme is presented to put the whole in perspective.

Introduction

> When in doubt, enlarge the quantum system. Then it is found that the division can be so made that moving it further makes very little difference to practical predictions. Indeed good taste and discretion, born of experience, allows us largely to forget, in most calculations, the instruments of observation.[1]
>
> The continuing dispute about quantum measurement theory is... between people who view with different degrees of concern or complacency the following fact: so long as the wave packet reduction is an essential component, and so long as we do not know exactly when and how it takes over from the Schroedinger equation, we do not have an exact and unuambiguous formulation of our most fundamental physical theory.[2] *J. S. Bell*

1 A Phenomenological Model

After discussing the measurement situation generally, we introduce the concept of the *pointer basis*, and then concentrate on the process which SE fails to describe: the step whereby the measurement's outcome is decided, the collapse. We introduce a simple model based on the dynamics of a random walk in probability space. This will serve as the phenomenological basis for MOM.

1.1 About measurements

To motivate the assumptions of this model, let us consider how measurements are usually thought to occur. The objects involved are divided into two sets, the micro(quantum)-systems and the macro(classical)-systems. Microsystems do not obey classical laws of physics: They evolve by SE like all quantum systems. Classical systems are detectors and state preparation devices: They are devoid of quantum ambiguities. The measurement process is as follows. A microsystem is prepared in a definite known state by use of a classical preparation apparatus. For self-consistency we may consider this to be the result of a previous measurement. The microsystem is then allowed to evolve for a finite period of time, as correlations are created between the microsystem and another system. The second system evolves, becoming correlated with another system, and so on ... Thus a chain of systems is obtained wherein each member's state is correlated perfectly with the state of the original microsystem: 'von Neumann's chain'. At some point, however, there is at least one member of the chain which is a classical system. It cannot partake in any quantum ambiguity, so it must make a choice among some set of classically allowed states. The classical apparatus comes to a definite arrangement, and the correlations induce a cascade along the chain which leaves all the systems in compatible states. The measurement has been performed. In the orthodox view, the particular chain of systems used is not important, nor the place of transition between microsystems and classical apparatus. What counts is that there be a microsystem at one end, a classical system at the other end, and the appropriate interactions between the chain elements so the correct correlations are established. The details of the process do not affect the probabilities for the various classically observable results and are therefore unphysical, unobservable.

Experimenters routinely link together such chains of systems in order to perform measurements. Nevertheless, there is a theoretical problem. In the transition region between quantum and classical behavior there must be at least one 'hybrid' system: It must interact quantum-mechanically in order to become correlated with the previous (quantum) system, but it must also make a choice among the various observable outcomes. This system must obey neither quantum nor classical laws of evolution. Its existence would verify the objectivity of the collapse process. But then it should be possible to devise an experiment to violate the statistical predictions of quantum theory, i.e. BP. If no such a system is ever identified and isolated, it could be that the transition from quantum to classical behavior is illusory, measurements are not well described by the above picture, and there is no collapse, *or* that this transition is absolute, there being only microsystems and detectors but no 'hybrids'. Either case is theoretically troublesome.

Assuming such hybrid systems do exist, what kinds of systems are they (i.e. which degrees of freedom are involved), what are their dynamics, and what deviations from standard quantum predictions should we expect? Here we assume the answer to the first question to be given: Certain systems are assumed to be special in this regard. With this information we postulate a conceptual model and detailed dynamics. From these we calculate what unusual results we should expect in ordinary measurements, where the quantum/classical transition does not play a major role. They are limited to small deviations from BP.

1.2 Pointer-basis situation

Simple measurements are most easily understood by invoking the existence of a *pointer basis*. This is a special basis for the small, or 'local' Hilbert space. A pure local state ψ can be written as

$$|\psi\rangle = \sum_{i=1}^{N} s_i |i\rangle \ ,$$

and the complete, or 'global state' Ψ as

$$|\Psi\rangle = \sum_{i,a=1}^{N,A} C_{ia} |i\rangle \otimes |a\rangle \ ,$$

where $|i\rangle$ is a pointer-basis element and $|a\rangle$ is a basis element in the Hilbert space of the environment degrees of freedom. The pointer-basis assumption is that the self energy of ψ and the interactions between the local system and environment are diagonal in the pointer basis. It must be emphasized, that in case the interactions *do not* commute with the self energy, there is *no* pointer basis: The validity and usefulness of this concept varies with the system. This is relevant to the task of experimenters. Their purpose is to set up a situation, where the system of interest can find its way onto one state from a set of states which comprise the set of possible experimental results. They will know that one of these states has been reached, and which one, by the fact that some part of the experimental apparatus, the *pointer(!)*, has reached one of the corresponding positions. This is the origin of the term, pointer basis. The apparatus self energy must be such, that once the pointer gets to one of these positions, it stays there long enough for the fact to be recorded elsewhere. If the self energy quickly drives it away, the moment will pass unnoticed, and will not constitute a practical measurement. The reason to introduce the pointer basis in the micro-system is the same: Its existence is a *prerequisite* for the *possibility* of any measurement. Below, we will simplify the dynamics even further by assuming that all self energies vanish, and that the interactions can be written as

$$\hat{H} = \hat{H}_{int} = \sum_{ia} H_{ia}|ia\rangle\langle ia| \ , \quad |ia\rangle \equiv |i\rangle \otimes |a\rangle \ .$$

For future use we define a *global state reduced density matrix* (GDM) $\hat{Q}$, which is the global density matrix reduced to a local-Hilbert space density matrix by tracing over the environment:

$$\hat{Q} = \sum_a \langle a|\Psi\Psi^\dagger|a\rangle = \sum_{ij} Q_{ij}|i\rangle\langle j| \ , \quad Q_{ij} = \sum_a C_{ia}C^*_{ja} \ . \tag{1}$$

These assumptions simplify the dynamics greatly while maintaining the essential features of common measurements. The outcome of such a measurement is one of the pointer-basis elements. How natural is the pointer-basis assumption? On the one hand, all physical systems violate it to some extent, since interactions depend on coordinates such as particle positions, whereas self-energies contain derivative terms(kinetic energy): The two do not commute. On the other hand, usually every effort is made to diminish the effect of the kinetic terms so that measurements are stable. Eventually every measurement outcome becomes undone by natural processes, but if it lasts long enough to be recorded, this is not a problem. The pointer-basis assumption is justified when considering standard measurement procedures.

1.3 Probability random walk

If the system of interest, which is undergoing a measurement, has interacted with its environment long enough (i.e. over one or more decoherence times), it can be represented by a density matrix which is diagonal in the pointer basis:

$$Q_{ij} \to Q_i\delta_{ij} \ .$$

The task for a model of measurements is to select one of these basis states as the outcome. We can do this in a simple way, by postulating that the system undergoes a *random walk*. This is the essence of Pearle's 'models for reduction', which are defined stochastically with no attempt to explain the process in a deeper way [3, Pearle]. This is a useful paradigm, a context for discussion of such models, which must add some mechanism, random or deterministic, to generate the random walk. Such a mechanism may lead to predictions which are in violation of BP, or otherwise contrary to standard quantum theory, providing a basis for experimental judgement. MOM, described below, *has* deviations from BP, while the basic Pearle model and the simple model described here are not specific enough to be tested experimentally.

Let us consider a two-level system first. The density matrix is represented by a single real number z between zero and one, which according to the Born rule, is the probability that one of the two states

is the measurement outcome. The random walk assumption is that, after each time interval δt, z will change to either $z + \delta z$ or $z - \delta z$, where δz is constant, with equal probability (*symmetry condition*). The only exception is that, if z reaches 0 or 1, the process stops, as the measurement outcome has been decided. In the ideal case, δz is a constant integer fraction of 1, and the initial value z is an element of the set $\{0, \delta z, 2\delta z, \ldots, 1 - 2\delta z, 1 - \delta z, 1\}$. This leads to the confirmation of the Born rule: The measurement outcomes occur with probabilities z and $1-z$. If either z or δz doesn't satisfy its condition, the probabilities will deviate from BP by at most δz. The average time before an outcome is reached is finite and proportional to $\delta t/(\delta z)^2$. There are some paths which do not terminate at either 0 or 1, but these are of measure zero. These simple results can be deduced from, for example, Feller[4]. In the case of a system with N states, the situation is similar but slightly trickier. The symmetry condition can still be maintained, but we replace δz with the average over $N - 1$ directions, $\overline{\delta z}$.

Given a small enough $\overline{\delta z}$, this crude model will reproduce BP for measurement outcomes. However, it is inadequate as a serious physical description of the physical processes involved in several ways. The system undergoing measurement is chosen arbitrarily. Unless this system is the last one in von Neumann's chain, the particular choice will affect the oucome probabilities, if $\overline{\delta z}$ varies from system to system. The choice for pointer basis is arbitrary, whereas it should be dictated by some features of the physical systems under consideration. The interactions of the system with its environment should determine the choice, and whether the pointer-basis assumption applies at all. The cause of the random changes in the system is unspecified. If there is no explanation for this, the mystery of the state-vector collapse is not solved but merely relocated. Is there really such a thing as *fundamentally* random behavior in the universe? The values of δt and $\overline{\delta z}$ are arbitrary. Thus they may change from system to system, and can take on values such that no experiments will reveal the probability deviations. A more detailed model could constrain or predict these numbers, and hence could be refuted or confirmed by experiments.

2 The Multiple Observer Model

MOM is based on both structural and dynamical changes to standard quantum theory: Here we present the structural and conceptual elements, leaving the dynamics for Part I below. It is assumed that there are *multiple observers*, each associated with a specific *cut*. To understand the cut, see Figure 1. First consider(a) the set $\mathcal{U}$ of all classical degrees of freedom. A cut(b) is an arbitrary partition of this set into an interior domain $\mathcal{D}$ and the exterior, the environment, which is its complement $\mathcal{U} - \mathcal{D}$. The interior degrees of freedom form the foundation for a Hilbert space $\mathcal{H}_\mathcal{D}$, of which the observer, the quantum state ψ is a member. The location of the cut is arbitrary, and it defines a set of classical degrees of freedom which is somehow special. MOM, is its present form, does not attempt to prescribe the choice of cuts, or explain why some are *active*, i.e. have quantum states ψ associated with them, and some are not. The usual quantum state of the universe is recovered(c) by allowing the interior of the cut to be equal to $\mathcal{U}$. In this work, we assume this cut to be active, so that the usual state in orthodox quantum theory is always present. In the absence of other active cuts, it would evolve normally, under SE. Any number of cuts may be active. The general relationship between two arbitrary cuts is depicted in (d,e and f): They may be disjoint, overlap partially, or may be contained one inside the other. If they are disjoint, they do not affect each other directly at all, as they share no classical degrees of freedom. Only if they overlap partially or completely will there be the potential for discrepancy between the information contained in the two states, and then their evolution will not be independent: I.e. they will 'interact'. Apparently, the simplest case that is non-trivial is (g), wherein there is the *global* state Ψ, which is always assumed to exist, and a *local* state ψ, associated with an active cut. This will be enough to describe a normal quantum measurement. It is possible to include more active cuts. For instance, results of EPR-type experiments may some day deviate from BP in such a way, that the simple picture, using just one local state and the global state, is

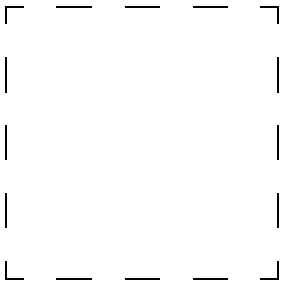
not able to describe the situation. In that case(h), we could introduce *two* local states, one associated with each detector array. The set of all possible numbers and arrangements of active cuts is enormous.

Let us now preview the results of MOM. If there is a single observer the evolution of its state vector is given as usual by SE: This is the standard quantum theory, minus collapse of the wavefunction. A single observer in isolation apparently experiences nothing but the uniform passage of time. If there are two observers A and B we take $\mathcal{D}_A \subset \mathcal{D}_B = \mathcal{U}$. This is the simplest case in which measurements can occur. The state ψ in the small Hilbert space acquires an effective Hamiltonian, but also jumps at discrete times. These jumps are associated with the tendency for the full Hamiltonian to 'decohere' the state ψ. The state Ψ in the large Hilbert space just sees the Hamiltonian including all interactions. In addition, however, both states experience a new kind of interaction, which occurs between observers and not between degrees of freedom: This is attractive, always tending to make the two states compatible. Eventually both ψ and Ψ fall into compatible states, eigenstates of the measured operator. This 'observable' is not arbitrary but is determined by normal interactions which cross the cut, i.e. involve classical degrees of freedom inside and outside of the cut domain $\mathcal{D}_A$. If there are three observers A, B, and C, their domains may overlap in a complicated way, or may be nested ($\mathcal{D}_A \subset \mathcal{D}_B \subset \mathcal{D}_C = \mathcal{U}$), or parallel ($\mathcal{D}_A \subset \mathcal{D}_C = \mathcal{U},\ \mathcal{D}_B \subset \mathcal{U},\ \mathcal{D}_A \cap \mathcal{D}_B = \emptyset$). The latter case may describe an ordinary measurement with two distinct detectors, such as EPR, Schroedinger's cat, or Wigner's friend. For simplicity we restrict ourselves to the two-observer case, as it already contains the elements needed to assess the merits of the model.

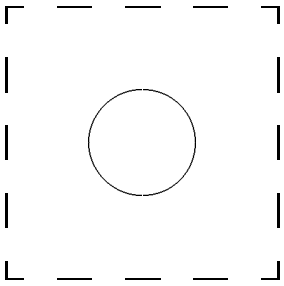
I: Dynamics

> I am, in fact, rather firmly convinced that the essential statistical character of contemporary quantum theory is solely to be ascribed to the fact that this (theory) operates with an incomplete description of physical systems... Assuming the success of efforts to accomplish a complete description, the statistical quantum theory would, within the framework of future physics, take an approximately analogous position to the statistical mechanics within the framework of classical mechanics. I am rather firmly convinced that the development of theoretical physics will be of this type, but the path will be lengthy and difficult. *Albert Einstein* [5, pp.666-672]
>
> ..., in quantum mechanics, we are not dealing with an arbitrary renunciation of a more detailed analysis of atomic phenomena, but with a recognition that such an analysis is *in principle* excluded ... As regards the specification of the conditions for any well-defined application of the formalism, it is moreover essential that the *whole experimental arrangement* be taken into account ... it is decisive to recognize that, *however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms* ... This crucial point ... implies the *impossibility of any sharp separation between the behavior of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear* ... As regards the specification of the conditions for any well-defined application of the formalism, it is moreover essential that the *whole experimental arrangement* be taken into account. *Niels Bohr* [5, pp.200-241]
>
> You believe in a dice-playing God and I in perfect laws in the world of things existing as real objects, which I try to grasp in a wildly speculative way. *Albert Einstein* [5, p.176]

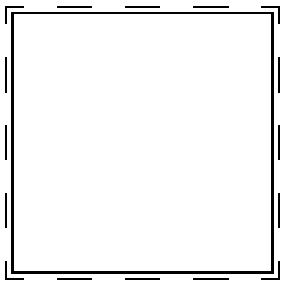
The dynamics of the various states is assumed to occur in discrete time, because of both theoretical and practical bias. On the theory side, we expect that time scales below a certain duration Δt, normally taken to be the Planck time, are not properly described by any theory of physics presently available, but that they are of primary importance in the measurement process. Therefore, since MOM is not to be considered as a fundamental theory of nature, it summarizes sub-Δt dynamics as discrete maps of the states, occurring during succesive cycles of duration Δt. On the practical side, it is often easier to implement evolution by difference, rather than differential equations. Also, chaotic dynamics, which are a *necessary* ingredient of MOM in the measurement regime, are easier to produce at low dimensionality: There are no chaotic continuous evolution equations for real dimension less than three. During each cycle, there is in addition to the usual action of the Hamiltonian, a decoherence-induced mapping of the local state vector(s), and a



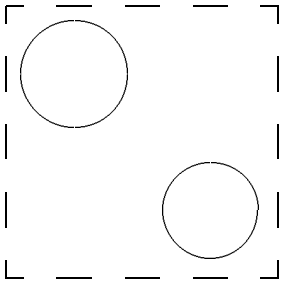
a) Set $\mathcal{U}$ of classical degrees of freedom.



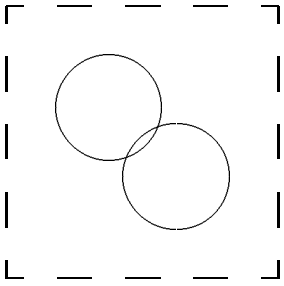
b) A domain $\mathcal{D}$, the interior of a cut. $\psi \epsilon \mathcal{H}_{\mathcal{D}}$



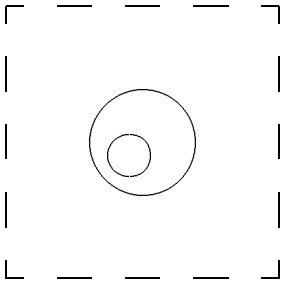
c) The global domain $\mathcal{U}$. $\Psi \epsilon \mathcal{H}_{\mathcal{U}}$



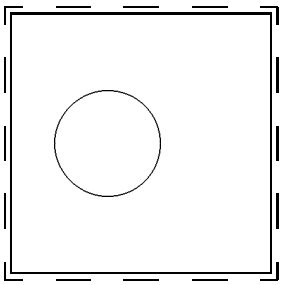
d) Two domains $\mathcal{D}_1$ and $\mathcal{D}_2$, $\mathcal{D}_1 \cap \mathcal{D}_2 = \emptyset$.



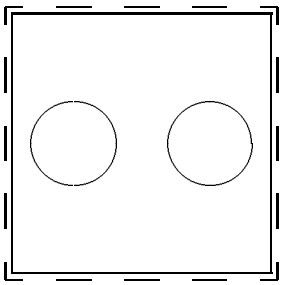
e) Two domains, $\mathcal{D}_1 \cap \mathcal{D}_2 \neq \emptyset$.



f) Two domains, $\mathcal{D}_1 \subset \mathcal{D}_2$.



g) Local and global domains $\mathcal{D}$ and $\mathcal{U}$, minimum needed for any measurement.



h) Three domains $\mathcal{D}_L$, $\mathcal{D}_R$, and $\mathcal{U}$, possibly needed to describe EPR-type measurements.

Figure 1: The observer/environment cut

universal attraction between all pairs of observers. In this section, we describe the different components of the dynamics. There is the usual Hamiltonian evolution, the chaotic map, and a new kind of interaction between different observers. Each part of the dynamics in turn plays its role in the *dynamical cycle*.

3 Effective Hamiltonian

It is reasonable to extract as much information as possible about the dynamics from the complete Hamiltonian of the system. This will help to maintain the distinction between ordinary physical properties of the system and new, model-related parameters. First we note that this global Hamiltonian is still applicable to Ψ. The effective Hamiltonian for ψ can be obtained as follows. We construct a projection operator in the global Hilbert space from ψ which is the product of a pure state operator over the local space times the identity over the environment space:

$$\mathcal{P} = \psi\psi^\dagger \otimes \sum_{a=1}^{A} |a\rangle\langle a| \ . \tag{2}$$

We then apply the standard Schroedinger evolution to this operator, using the full Hamiltonian, for a time interval Δt, and keep only those terms of order Δt. The result is that ψ changes but the general, factorizable form prescribed above does not. We can then uniquely define an operator which will reproduce this evolution when inserted into SE for ψ. The result is the basis-independent effective Hamiltonian, or self-energy:

$$\hat{H}_{eff} = \sum_{i=1}^{N} V_i |i\rangle\langle i| \ , \quad V_i = \frac{1}{A}\sum_{a=1}^{A} H_{ia} \ .$$

In this work this is assumed to vanish identically. It is constant in time if the global Hamiltonian is constant as usual. Note that this was derived by making the *no-information* assumption: The local state has no information whatsoever about the environment. If the same procedure were carried out, starting from the global state, $\hat{H}_{eff}$ might be different. In that case, it might not be possible for the global and local states to come together in compatible states. In the following, we assume this is not a problem.

4 Chaotic Dynamical Map

Here we discuss the aspect of the dynamics which leads to chaotic behavior in certain cases. This stems from the decohering interactions assumed present in the Hamiltonian, as is represented by a discrete-time map. First we describe the decoherence effects, which would be present in a standard description of the local subsystem, and then define the map with a somewhat more general approach. The general behavior of the system under the mapping is briefly sketched.

4.1 Decoherence

Let us consider the terms of order $(\Delta t)^2$, which were ignored above. After subtracting the self-energy contributions (which will be assumed to vanish from now on), we are left with terms which violate the assumption of factorizability. In other words, the interactions between the local degrees of freedom and the environment tend to decohere the local state ψ. But this is contrary to the basic assumption that ψ remains pure at all times. It is natural to postulate that some mechanism acts to prevent decoherence, opposing the effect of the interactions. Although the details of such a mechanism are beyond the scope of this model, we can make some observations. First, this effect vanishes if an attempt is made to describe it as occurring continuously in time: This is because it is absent to order Δt. Hence we will describe it as occurring at

discrete time intervals. Second, the function describing the amount of decoherence of ψ is mathematically identical, under suitable symmetry assumptions, to what one might guess from first principles(see below). In particular, the decohering effect vanishes near the pointer-basis states. Thus we are led to define a second element of the dynamics for ψ, a discrete-time map, which is chaotic if the decohering interactions are strong enough. These maps were previously investigated numerically[6]: In the chaotic regime, in the absence of competing dynamical effects, they lead to highly symmetrical distributions for ψ which are concentrated near the pointer-basis states. In effect, ψ undergoes a process of *chaotic decoherence* over time, as opposed to the usual decoherence which is associated with an ensemble of systems at a given time. Note that the global state does not suffer decoherence in any way from the Hamiltonian, and hence is not affected by any such dynamical map.

Specifically, let us begin at time $t = 0$ with a global density matrix $\hat{D}$, derived from ψ, which is a multiple of the projector $\mathcal{P}$ defined above(2). This $\hat{D}$ will evolve, under the usual SE, into $\hat{D}(t)$, and will at time Δt no longer be a multiple of a projector, but will have eigenvalues other than 0 and 1. At time Δt we define a small decoherence parameter $\mathcal{Z}$, such that, when $\hat{D}(\Delta t)$ is reduced to a local density matrix, its largest eigenvalue is $1 - \mathcal{Z}$. We find $\mathcal{Z}$ in terms of the Hamiltonian:

$$\mathcal{Z} = \frac{1}{2}(\frac{\Delta t}{\hbar})^2 \sum_{ij} X_{ij} p_i p_j \ , \quad X_{ij} = \frac{1}{A} \sum_a (H_{ia} - H_{ja})^2 \ , \quad p_i = |s_i|^2 \ . \tag{3}$$

A more symmetric form is achieved by making the assumption

$$X_{ij} \to (1 - \delta_{ij}) E^2 \ ,$$

$$\mathcal{Z} \to \frac{1}{2}(\frac{E\Delta t}{\hbar})^2 \sum_i p_i (1 - p_i) \ . \tag{4}$$

We can then solve the eigenvalue problem to find the eigenstate $|\phi\rangle$ corresponding to $\mathcal{Z}$. If, approximately, $|\phi\rangle \approx |\psi\rangle + |\psi_\perp\rangle$, then

$$|\psi_\perp\rangle = -\frac{1}{4} |\nabla_{\psi,\perp} \mathcal{Z}\rangle : \text{not normalized} \ , \tag{5}$$

where $\nabla_{\psi,\perp}$ is the component of the gradient operator in the local Hilbert space normal to $|\psi\rangle$. We define the map to be a rotation of $|\psi\rangle$ in the direction of $|\psi_\perp\rangle$, through an angle equal to the norm of $|\psi_\perp\rangle$. If the decohering interactions are smaller than $\Delta E \equiv \hbar/\Delta t$, $\mathcal{Z}$ remains small, and this procedure does not lead to the required chaotic map. For the map to be truly chaotic, we require $t_E \equiv \hbar/E \ll \Delta t$: The decoherence time must be much less than the undetermined cyclic time scale. In what follows we will use $\mathcal{Z}$ as defined above, even when it is *not* small.

4.2 Abstract definition of the decoherence map

We first define $\mathcal{Z}$, the generating function for the map, using a physically-motivated set of operators. Then we define the map in the general case, note the result for the case, $N = 2$, and continue for larger values of N.

The pointer basis is assumed to have special significance during the measurement process, so we will use it as the basis in which to write the state vector ψ. For convenience, we will use a mixed braket-vector notation:

$$\begin{aligned} \psi \quad &: \quad |\psi\rangle = \sum_{i=1}^{N} \psi_i |i\rangle \ , \\ &: \quad \hat{R} = \sum_{i=1}^{N} R_i \hat{R}_i \ , \end{aligned}$$

$$\psi_i \equiv R_i e^{i\theta_i} \ , \ \ p_i = \|\psi_i\|^2 = R_i^2 \ , \ \ R_i \in \Re \ .$$

The basic operators, diagonal in this basis are $\hat{\theta}_i = |i\rangle\langle i|$, and their variances in the state are

$$\Delta_i^2 \equiv \langle\psi|\hat{\theta}_i^2|\psi\rangle - \|\langle\psi|\hat{\theta}_i|\psi\rangle\|^2 = p_i(1-p_i) \ .$$

A single operator is fine in the $N = 2$ case, but theoretical bias suggests that the relevant set of operators, in the measurement of a larger system, should be a *complete set of commuting operators*. We will use the identity operator, with zero variance, and the set

$$\hat{K}_c = \sum_i q_{ci}\hat{\theta}_i \ , \ \ c = 1, 2, \ldots, N-1 \ .$$

The variances and $\mathcal{Z}$ become

$$\Delta_c^2 = \sum_i \|q_{ci}\|^2 p_i - \sum_{ij} q_{ci}^* q_{cj} p_i p_j \ ,$$

$$\mathcal{Z} \equiv \sum_c \Delta_c^2 = \sum_i K_{ii} p_i - \sum_{ij} K_{ij} p_i p_j \ ,$$

$$K_{ij} \equiv \sum_c q_{ci}^* q_{cj} \ .$$

All operators in the set already commute. The condition of completeness is $K_{ij} = K_i \delta_{ij}$. For further symmetry, we set $K_i = K$, and the function $\mathcal{Z}$ attains the form:

$$\mathcal{Z} = K \sum_i p_i(1-p_i) \ . \tag{6}$$

Any map generated from this function will have a symmetric distribution, if no bias is introduced from another source. Also, the minima(i.e. zeroes) of $\mathcal{Z}$ are at the pointer-basis states.

The function $\mathcal{Z}$ does not depend on the phases θ_i, so the vector notation is convenient. I.e., only the R_i will be affected by the map, including changes of sign. We define the gradient of $\mathcal{Z}$ in the space of R_i, then the component of the gradient which is tangent to the unit sphere.

$$\frac{\partial \mathcal{Z}}{\partial R_i} = 2KR_i(1-2R_i^2) \ ,$$

$$\vec{\nabla}\mathcal{Z} = \sum_i \hat{R}_i \frac{\partial \mathcal{Z}}{\partial R_i} = 2K \sum_i R_i(1-2R_i^2)\hat{R}_i \ ,$$

$$\hat{R} \cdot \vec{\nabla}\mathcal{Z} = 2K \sum_i R_i^2(1-2R_i^2) \ ,$$

$$\begin{aligned} \vec{\nabla}_\perp \mathcal{Z} &\equiv \vec{\nabla}\mathcal{Z} - \vec{\nabla}_\parallel \mathcal{Z} \ , \\ &= \vec{\nabla}\mathcal{Z} - \hat{R}(\hat{R} \cdot \vec{\nabla}\mathcal{Z}) \ , \\ &= 4K \sum_i R_i(S_4 - R_i^2)\hat{R}_i \ , \ \ S_4 = \sum_j R_j^4 \ . \end{aligned}$$

This is zero if and only if all non-zero components R_i have the same magnitude. This occurs only at the pointer-basis states, and at other, partly symmetric points: These are the fixed points of the map. In the case of chaotic maps, when $K \gg 1$, the pointer-basis states are unstable or marginally unstable, while the others are absolutely unstable. Once the tangent gradient has been defined, we produce a unit-vector $\hat{G}$ in

the same direction, and define the map as a rotation. The plane of rotation is spanned by $\hat{R}$ and $\hat{G}$, and the angle by which the state $\hat{R}$ is to be rotated is related to the magnitude of the tangent gradient.

$$\begin{aligned} \hat{G} &\equiv \|\vec{\nabla}_\perp \mathcal{Z}\|^{-1} \vec{\nabla}_\perp \mathcal{Z} \ , \\ &= (S_6 - S_4^2)^{-1/2} \sum_i R_i (S_4 - R_i^2) \hat{R}_i \ , \quad S_6 = \sum_j R_j^6 \ . \end{aligned} \tag{7}$$

$$\text{map} : \hat{R} \to \hat{R}' = \hat{R} \cos\theta + \hat{G} \sin\theta \ , \quad \theta \equiv |\vec{\nabla}_\perp \mathcal{Z}\| \ . \tag{8}$$

The constant K determines the strength of the map. The fixed points are just the pointer-basis states and the other places where the tangent gradient vanishes, as noted above.

4.3 Mapping results

In case $N = 2$, there is only one independent variable in the map. Let this be the angle $\phi \epsilon [0, 4\pi]$:

$$R_1^2 \equiv \sin^2(\phi/2) \ , \quad R_2^2 = \cos^2(\phi/2) \ , \quad \mathcal{Z} = (K/2) \sin^2\phi \ .$$

The map becomes

$$\phi \to \phi' = \phi + k \cos\phi \sin\phi \equiv f(\phi) \ ,$$

where the new constant k replaces K, and is also arbitrary and real. The derivative of f is

$$df/d\phi = 1 + k[\cos^2\phi - \sin^2\phi] \ .$$

We are most interested in the behavior near the pointer-basis element, i.e. for $\phi \ll 1$:

$$df/d\phi \approx 1 + k \ .$$

A 'hopping' method was used[6] to obtain the Liapunov exponent of the map and the invariant distribution in the chaotic regime. The Liapunov exponent becomes positive for large enough values of the map strength k, and grows like the log of k. The invariant distribution starts as a small island around the maximum of the function $\mathcal{Z}$, and grows to encompass the entire space for larger values of k. This distribution becomes more uniform as k is increased. Similar results were obtained for $N = 2, 3, 4$.

5 The Shift Map

This mapping is used to change the form and statistical properties of the discrete map induced on the local Hilbert space $\mathcal{H}$ by the decohering interactions, which we will call the *bare* map. The bare map has some of the properties which we require for the model of quantum measurement processes, but it has some shortcomings. Namely, it leads to a distribution for the local state which is uniform over a subspace of $\mathcal{H}$, determined by the initial state: Any point on this subspace is visited by the local state just as often. This means the pointer-basis states have no privileged status, and the random probability walk of the global state will not yield the correct probabilities for outcomes of measurements, even approximately(§7). But this distribution can be altered essentially at will, by use of the appropriate *shift map*.

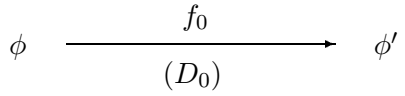


Figure 2: Bare map

5.1 Effective map and invariant density

The bare map represents the effect of the decohering interactions on the local state, and is executed after every cycle Δt. The map is represented by Figure 2, where $\phi \epsilon \mathcal{H}$, $\phi' = f_0(\phi)$, f_0 is the bare map, and D_0 is the distribution over $\mathcal{H}$ associated with f_0.

To change the map without altering its fixed points, we can use another function on $\mathcal{H}$, which must have an inverse: an isomorphism g. The resulting complete or effective map f is represented by Figure 3, where $\psi \epsilon \mathcal{H}$, $\psi' = f(\psi) = g \circ f_0 \circ g^{-1}(\psi)$, and D is the true distribution over $\mathcal{H}$, given by

$$D(\psi) = D_0(\phi) \| \nabla_{\psi,\perp} g(\phi) \|^{-1}, \;\; \phi = g^{-1}(\psi) \; , \tag{9}$$

where the gradient of g is with respect to ψ, tangent to $\mathcal{H}$ because of the constraint $\|\psi\|^2 = 1$, and the double bars around ∇ indicate taking a determinant of this multi-dimensional derivative.

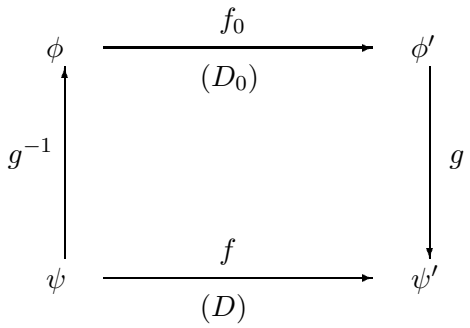


Figure 3: Effective map

One can see the effect of g by imagining the distribution D_0 is represented by a multitude of points ϕ, and allowing each of them to be moved by g to new locations. The resulting distribution is D. Now, we want the distribution to be more sharply concentrated near the pointer-basis states, where the decoherence generating function $\mathcal{Z}$ is minimum (§4). Also, we want to define g to depend only on $\mathcal{Z}$, without introducing any unnecessary parameters. We are led to adopt the following prescription. The function g is the result of integrating an ordinary differential equation(with respect to a dummy variable s) over a unit interval, with ϕ as initial value and ψ as final value. The equation is

$$\frac{d}{ds}|X\rangle = -|\nabla_{\psi,\perp} \mathcal{Z}\rangle \; , \;\; |X(s=0)\rangle = |\phi\rangle \; , \;\; |X(s=1)\rangle = |\psi\rangle \; . \tag{10}$$

This form will ensure that D is concentrated near the minima of $\mathcal{Z}$ when $\mathcal{Z}$ itself is large, i.e. when the decohering interactions are strong, and the bare map is truly chaotic. However, in the limit of small $\mathcal{Z}$, the inverse map g^{-1} becomes identical to f_0, so $f \to f_0$. Therefore we will mostly consider the strongly chaotic regime in what follows.

5.2 Specific results for N=2

In the special case $N = 2$, we can derive relatively simple specific formulas to help us understand the effect of the shift map g: These were useful in numerical work[6]. In the cases of interest, $\mathcal{Z}$ has such symmetry that both f_0 and g are one-dimensional maps. Let x be the variable before the shift map(like ϕ), and z the variable after the map(like ψ). We consider the behavior of the maps near a fixed point(a pointer-basis state) which we assume to be zero: $f_0(0) = g(0) = 0$. Near this state, $g(x) \approx ax$, where a is arbitrary. The result is that if $f_0(x) \approx kx$:, then $f(z) \approx kz$. I.e., the map behavior near the pointer-basis states, and the capture process discussed in §7.2, are unchanged by the shift map.

Now let us take the specific, standard form of the decoherence-generating function $\mathcal{Z}$, and derive the actual maps and distributions.

$$\mathcal{Z} = (1/2)R^2 p(1-p), \quad R \equiv E\Delta t/\hbar \ ,$$

$$|\nabla_{\psi,\perp}\mathcal{Z}\rangle = R^2\sqrt{p(1-p)}(\sqrt{1-p}|1\rangle - \sqrt{p}|2\rangle) \ ,$$

$$|\psi\rangle \equiv \sqrt{p}|1\rangle + \sqrt{1-p}|2\rangle \ .$$

Using the general form for the shift flow equation:

$$\frac{d}{ds}|\psi\rangle = -|\nabla_{\psi,\perp}\mathcal{Z}\rangle \ .$$

we get

$$\frac{dp}{ds} = -\lambda(1-2p)p(1-p) \ , \ \lambda = 2R^2 \ .$$

We are interested most in the behavior near the pointer-basis states, e.g. near $p = 0$. There, this becomes $dp/ds \simeq -\lambda p$, so the shift map is linear. For arbitrary integration limit s(normally, $s = +1$ for the direct map, $s = -1$ for the inverse map), the map is $p_0 \to p(s) = e^{-\lambda s}p$.

We can solve the equation explicitly to obtain the shift map for any p, with the aid of some variable changes:

$$p \equiv \sin^2\theta, \quad z \equiv \tanh^{-1}(\cos\varphi), \quad \varphi \equiv 4\theta,$$

$$\frac{dz}{ds} = \lambda/2 \ , \quad z(s) = z_0 + R^2 s \ ,$$

$$p(s) = \sin^2(\frac{1}{4}\cos^{-1}\tanh(z_0 + R^2 s)) \ .$$

We can also solve for the bare map explicitly. In terms of θ, it is defined as a rotation in the direction of the gradient $|\nabla_{\psi,\perp}\mathcal{Z}\rangle$, through an angle of magnitude $\| \ |\nabla_\perp \mathcal{Z}\rangle \ \|$.

$$\begin{aligned} \Delta\theta &= R^2 \| \cos 2\theta \| (\frac{1}{4}\sin^2 2\theta)^{1/2} \ , \\ &\simeq R^2\theta \ , \ \theta \simeq 0 \ . \end{aligned}$$

I.e., we get linear behavior:

$$\begin{aligned} \Delta\theta &\simeq R^2\theta \ , \\ \Delta p &\simeq (1+R^2)^2 p \simeq R^4 p \ , \ \text{in the chaotic regime.} \end{aligned}$$

The variable φ is the most transparent:

$$\varphi \to \varphi' = \varphi + R^2 \sin\varphi \ . \tag{11}$$

The distribution is nearly constant over φ, so for p is approximately

$$D(p) = \frac{2}{\pi}/\sqrt{p(1-p)}\ .$$

In terms of the transformed variable p', the shift map changes this to

$$D'(p') = K\frac{2}{\pi}/\sqrt{p'(1-p')}\ ,$$

$$K \equiv \frac{\sqrt{1-\rho^2}}{1-\rho(1-8p'(1-p'))}\ ,\quad \rho \equiv \tanh R^2 s\ .$$

We concentrate on the factor K. Since we are in the chaotic regime by assumption, $R^2 s \gg 1$, so $\rho \simeq 1 - 2e^{-2R^2 s}$. The factor becomes

$$K \simeq e^{R^2 s}/(1 + e^{2R^2 s} 4p'(1-p'))\ .$$

This is about one when $p' \simeq \bar{p} = (1/4)e^{-R^2 s}$, a very small value. Defining $p' \equiv a\bar{p}$, the factor K behaves thus:

$$\begin{aligned} a \ll e^{-R^2 s} &: K \simeq e^{R^2 s} \gg 1\ , \\ a \simeq e^{-R^2 s} &: K \simeq (1/2)e^{R^2 s} \gg 1\ , \\ a = 1 &: K = 1\ , \\ a \gg 1 &: K \simeq 1/a \ll 1\ . \end{aligned}$$

The net result is that, in the chaotic regime, the shift map causes the distribution over p to become severely concentrated near the pointer-basis states, as required by §7. The spread about each of these states should be no more than $\bar{p}$, which decreases *exponentially* with increasing decohering interaction strength.

Finally we summarize the explicit forms of the maps for the $N = 2$ case, with $\mathcal{Z}$ having the standard form and $s > 0$:

$$f = g \circ f_0 \circ g^{-1} : \varphi \to \varphi'\ , \tag{12}$$

$$\begin{aligned} g^{-1} : \varphi &\to \varphi_1 = \cos^{-1}\tanh(z - R^2 s)\ , \\ f_0 : \varphi_1 &\to \varphi_2 = \varphi_1 + R^2 \sin\varphi_1\ , \\ g : \varphi_2 &\to \varphi' = \cos^{-1}\tanh(z_2 + R^2 s)\ , \end{aligned}$$

where z and z_2 correspond to φ and φ_2, respectively.

6 Observer Interactions

This element of the dynamics is completely new, consisting of an attraction between pairs of states which, in the absence of other effects, results in the following. The global state Ψ tends toward a form which is factored between local and environment degrees of freedom. That factor containing the local information is the same as the local state ψ. Hence there is effectively only one description of the system, Ψ. However, this is true only asymptotically for large times, so that at any finite time there remains some information in ψ which differs slightly from Ψ. Symbolically, we have

$$|\psi\rangle \to |\psi'\rangle\ ,\ \ |\Psi\rangle \to |\Psi'\rangle = |\psi'\rangle \otimes \sum_{a=1}^{A} C_a |a\rangle\ . \tag{13}$$

6.1 Summary of relevant features

For simplicity we describe this effect by a discrete-time map which is geometrically motivated, so that only the strength of the attraction must be defined, given the two states involved. The result on Ψ is

$$|\Psi\rangle \to |\Psi'\rangle = A_1 \mathcal{P}|\Psi\rangle + A_2(1-\mathcal{P})|\Psi\rangle \ , \tag{14}$$

$$A_1 = 1+\alpha(1-z) \ , \ A_2 = 1-\alpha z(2+\alpha(1-z)) \ , \ z = \langle\psi|\hat{Q}|\psi\rangle \ ,$$

where $\mathcal{P}$ and $\hat{Q}$ are defined above(2,1). Here α represents the strength of the map, and is a positive real function of z, the overlap between ψ and Ψ. For the effect on ψ we have

$$|\psi\rangle \to |\psi'\rangle = a\hat{Q}|\psi\rangle + b(1-\frac{z}{y}\hat{Q})|\psi\rangle \ , \tag{15}$$

$$y = \langle\psi|\hat{Q}^2|\psi\rangle \ , \ \ a^2 = (1-b^2(1-\frac{z^2}{y}))/y \ .$$

Here the strength parameter is taken as b, between 0 and 1. The discrete time scale Δt is the same as above. We choose to make the strength of the attraction map on one state due to the other state depend only on the dimensionality of the two Hilbert spaces, and on the overlap z. Below we will see this ensures that if additional, irrelevant degrees of freedom are added to the system, there will be no significant effect on the dynamics. There, we also show, that the relative strength of the two maps should equal the ratio of the dimensionalities of the Hilbert spaces. This interaction has some formal similarities with gravitation between massive bodies: It acts universally between pairs of objects (state vectors), it is attractive, and its action on a given object depends on a measure of the object's 'inertia', here measured by the dimensionality. This helps one to imagine the behavior of the state vectors under the influence of these, otherwise novel, interactions.

6.2 Abstract definition of the attraction map

We consider the effect of one state on another state. First we take both states to be in the same Hilbert space, and then generalize by using a density matrix to represent the agent state. Finally we consider the stability of the map: If degrees of freedom are added to one or both states, how is the map affected? In the process, we will discover the way the *effective* strength of the map depends on the dimensionality of the Hilbert spaces.

Action of state on state

In this case, both states belong to the same Hilbert space. It is most convenient to use the projector $\hat{P} = |B\rangle\langle B|$ to represent the state B, which acts on state A. The map is defined as

$$|A\rangle \to |A'\rangle = a\hat{P}|A\rangle + b(1-\hat{P})|A\rangle \ , \tag{16}$$

$$a^2 = [1-b^2(1-z)]/z \ ,$$

where z is the *overlap* of A and B,

$$z = \langle A|\hat{P}|A\rangle = \|\langle A|B\rangle\|^2 \ ,$$

which is zero if they are orthogonal, and one if they coincide. For $b=1$, the map is weakest ($|A'\rangle = |A\rangle$), while for $b=0$, it is strongest ($|A'\rangle = |B\rangle$). The fixed-point condition of §7.1, where the probability walk of the global state is constrained to maintain BP, suggests we define the constant β:

$$1-b^2 \equiv z\beta \ .$$

The map now takes the form

$$|A'\rangle = \sqrt{1+\beta(1-z)}\hat{P}|A\rangle + \sqrt{1-\beta z}(1-\hat{P})|A\rangle \ . \tag{17}$$

The value $\beta = 0$ gives the weakest map, the identity. The maximum value of β^2 is found by constraining the square roots to be real:

$$\begin{aligned} (1+\beta(1-z) \geq 0) \quad &\Rightarrow \quad (\beta \geq -1/(1-z) \geq -1) \ , \\ (1-\beta z \geq 0) \quad &\Rightarrow \quad (\beta \leq 1/z \leq +1) \ . \end{aligned}$$

This gives $-1 \leq \beta \leq +1$. The variable of interest is z. It evolves according to

$$z \rightarrow z' = z + \beta z(1-z) \equiv f_\beta(z) \ , \tag{18}$$

$$\begin{aligned} df_\beta/dz \quad &= \quad 1+\beta(1-2z) \ , \\ &\stackrel{z\rightarrow 0}{\longrightarrow} \quad 1+\beta \ , \\ &\stackrel{z\rightarrow 1}{\longrightarrow} \quad 1-\beta \ , \end{aligned}$$

which shows that, for $\beta \rightarrow -\beta$, the effect is reversed: I.e. the *attraction* becomes *repulsion*. For $\beta = 0$, the map is trivial: $z' = z$. For $\beta = \pm 1$, the map is maximally attracting or repulsing, especially near $z = 1$ and 0, respectively.

Action of density matrix on state

In general the two states involved are not in the same Hilbert space. Consider the global and local states of the standard measurement scenario. The local state can be promoted to a density matrix in the global Hilbert space, or a projection operator with trace bigger than one. The global state can be reduced to a density matrix in the local Hilbert space, in general not a pure-state projector. In either case, the appropriate generalization of the previous section involves defining the effect of a density matrix $\hat{Q}$ on a pure state ψ. Because $\hat{Q}$ is not a pure-state projector, $\hat{Q}^2 \neq \hat{Q}$, so the algebra is slightly different. A convenient definition of the map is

$$|\psi\rangle \rightarrow |\psi'\rangle = a\hat{Q}|\psi\rangle + b(1-\frac{z}{\Gamma}\hat{Q})|\psi\rangle \ , \tag{19}$$

$$z = \langle\psi|\hat{Q}|\psi\rangle \ , \ \ \Gamma = \langle\psi|\hat{Q}^2|\psi\rangle \ , \ \ a^2 = [1-b^2(1-z^2/\Gamma)]/\Gamma \ .$$

In the same way as above, we define the strength parameter β. The map becomes

$$|\psi'\rangle = \frac{z}{\Gamma}\sqrt{1+\beta\frac{z}{\Gamma}(1-z)}\hat{Q}|\psi\rangle + \sqrt{1-\beta z}(1-\frac{z}{\Gamma}\hat{Q})|\psi\rangle \ . \tag{20}$$

It is of interest to note the behavior of the local state under this map, when the density matrix is close to a pure-state projector. Let $\hat{Q}$ and $|\psi\rangle$ be written in terms of a basis which includes this state as the member with label $i = \mu$, then define

$$Q \equiv \sum_{i\neq\mu} Q_{ii} \ll 1 \ , \ \ Q_{\mu\mu} = 1-Q \ ,$$

$$p \equiv \sum_{i\neq\mu} \|\psi_i\|^2 \ , \ \ \|\psi_\mu\|^2 = 1-p \ ,$$

$$Q \circ p \equiv \sum_{i \neq \mu} Q_{ii} \|\psi_i\|^2 \ll 1 \ .$$

First consider the case $Q = 0$, when the density matrix is a pure-state projector. For small p, the local state almost coincides with that state. The map becomes, for arbitrary p,

$$\begin{aligned} p \to p' \quad &= \quad (1-\beta)p + \beta p^2 \ , \\ &\stackrel{\beta \to 0}{\longrightarrow} \quad p \ , \\ &\stackrel{\beta \to +1}{\longrightarrow} \quad p^2 \ , \\ &\stackrel{\beta \to -1}{\longrightarrow} \quad 2p - p^2 \approx 2p \ . \end{aligned}$$

We will consider only $\beta \geq 0$, except in §9.3. When $\beta = +1$, this will overcome any linear map, such as the decoherence map above, even in the chaotic regime. We use that fact in §7.2, when considering the capture process, whereby the local state is held near a pointer-basis state by the GDM. If both Q and p are small but non-zero, the map is, to leading order,

$$\begin{aligned} p' \quad &\approx \quad (1-\beta)p + [\beta Q p + 2\sqrt{1-\beta}(1-\sqrt{1-\beta})Q \circ p] \ , \\ &\stackrel{\beta \to 0}{\longrightarrow} \quad p \ , \\ &\stackrel{\beta \to +1}{\longrightarrow} \quad Qp \ . \end{aligned} \tag{21}$$

This form will help us put a bound on Q, for capture to occur.

Strength parameter

The question, what value should β take, is related to whether a projector or a density matrix should be used to define the map. Above we used a projector to represent the local state, boosted up to the bigger Hilbert space of the global state. This had a large trace, but can be rescaled down to a density matrix with trace one. Consider a map defined by the matrix $\hat{M}$, and change the map by multiplying $\hat{M}$ by a constant, and possibly also changing β:

$$\begin{aligned} \hat{M} \quad &\to \quad \hat{M}' = k\hat{M} \ , \\ \beta \quad &\to \quad \beta' \ , \\ z \quad &\to \quad z' = kz \ , \\ \Gamma \quad &\to \quad \Gamma' = k^2\Gamma \ , \\ a \quad &\to \quad a' = k^{-1}\sqrt{[1-(b')^2(1-z^2/\Gamma)]/\Gamma} \ , \\ b \quad &\to \quad b' = \sqrt{1-k\beta' z} \ , \\ |\psi'\rangle \quad &\to \quad |\psi'\rangle' = ka'\hat{M}|\psi\rangle + b'(1 - \frac{z}{\Gamma}\hat{M})|\psi\rangle \ . \end{aligned}$$

We can regain the original map, if we let $k\beta' = \beta$. Since the trace of $\hat{M}$ has been multiplied by k, this suggests

$$\beta \mathrm{Trace}(\hat{U}) \equiv \beta_0 : \mathrm{constant} \ . \tag{22}$$

Here $\hat{U}$ is whatever operator we want to use to define the map, and the constant β_0 will be taken to be equal to one, which will make the maps as strong as possible.

Now we can see what this means, in terms of the dimensionality of the Hilbert spaces. If both states are from the same space, then the map is maximally strong, with $\beta = 1$. If the agent state is from a

smaller space, then assuming a projector is used instead of the boosted density matrix, $\beta = 1/R \ll 1$, where R is the ratio of the numbers of linearly independent states in the two Hilbert spaces (i.e. their dimensionality). In practice, the map is defined more naturally in terms of the projector, so this means the value of β appearing in the equations above will be small, and the local state has only a weak effect on the global state. If the agent state is from a larger space, then also $\beta = 1$, because the reduced density matrix used above to define the map already has trace equal to one, so the global state affects the local one strongly.

General case and stability

Now we want to define the map in general, and see what happens, when degrees of freedom are added somewhere. We distinguish between the mapped state ψ_B on domain B, to which the map is being applied, and the agent state ψ_A on domain A, which defines the map. The sets A and B will be arbitrary. We define their intersection $C = A \cap B$, the non-overlapping parts $\tilde{A} = A - C$ and $\tilde{B} = B - C$, and their common environment $\tilde{U} = U - (A \cup B)$, where U is the universe. Note that the simplified cases considered so far were first, $A = B$ or *state on state*, and then $A \subset B$ or *density matrix on state*. The effect of ψ_A on ψ_B is obtained as follows. We first form $\hat{D}_C(\psi_A)$, the reduced density matrix in the space of C obtained from ψ_A. We then extend this to a density matrix $\hat{D}_B(\psi_A)$ on the space of B, by first forming a projector and then normalizing it to unit trace. In this normalization, the factor β is introduced, which is the ratio of the dimension of the space of C to that of the space of B. This is always less than or equal to one: It controls the strength of the attraction map. The map is now defined as in the previous section, as a density matrix acting on state. Now consider adding degrees of freedom to the various domains.

Add to $\tilde{A}$: In this case, these are traced over while forming the reduced density matrix $\hat{D}_C$. If the 'new' state ψ_A still contains no correlation between the new degrees of freedom and C, then $\hat{D}_C$ does not change. If some correlation is included between the new degrees of freedom and C, this has an effect on $\hat{D}_C$, typically making it less coherent. Since new information has been added, this is not incorrect. In any event, the factor β is unchanged.

Add to $\tilde{B}$: In this case, the factor β is affected directly: It decreases, making the map weaker. The 'inertia' of state ψ_B has become greater.

Add to C: In this case, we have added the same degrees of freedom to both A and B, so they are more nearly the same size. The factor β increases, reflecting this, unless it is already equal to one.

Add to $\tilde{U}$: In this case the map is completely unaffected by construction.

In conclusion, we have defined the universal attraction map, which represents the influence of one state upon another, and serves to bring their information content into agreement. There is strong indication, that the agent state should be represented by a trace-one operator, acting with universal strength β_0, which we let equal one. In particular, this means that if one of the states is from a smaller Hilbert space, it will have a weaker influence on the other. The mapped state should always be represented by a pure state in its natural Hilbert space, not including spurious degrees of freedom, about which there is no information. Finally, we note that this is a *consensus-building* map, which acts to decrease disagreement between the two states about any degrees of freedom.

7 Collapse

Here we consider the basic collapse process as described by the dynamics of MOM. More details, enabling us to describe an actual measurement from beginning to end, will be supplied in §8. We have described

three dynamical elements: the Hamiltonian, the decoherence map, and the attraction map. They do not act in isolation, but together, to define the actual dynamics of ψ and Ψ. The details of how they are combined should depend on an underlying theory which we do not yet know. We assume that they act sequentially within each cyclic time interval Δt: First the Hamiltonian will act (including all decoherence effects) and then the universal attraction.

To understand the role of collapse within the overall process, a brief sketch of a measurement is necessary. Before a measurement begins, the two states are almost in agreement at one of the pointer-basis states: The global state is factorized. In the preparation phase of every measurement there are, in addition to the pointer-basis interactions, other interactions which are not of that type, which we will call *active*. In the case of a controlled experiment, these interactions serve to manipulate or prepare the system into an appropriate superposition or mixture of pointer-basis states whose properties will be measured. They move the global state Ψ away from its original pointer-basis state, introducing correlations between the local and environment degrees of freedom: The local state ψ is basically unaffected. When the active interactions are effectively over there are left only the pointer-basis interactions. What follows is a period of time wherein ψ undergoes chaotic evolution(if the decohering interactions are strong enough) and Ψ acts as if under the influence of a random walk, while maintaining on average the probabilities associated with each pointer-basis state, which were generated by the active interactions. Finally Ψ comes close enough to a pointer-basis state to capture ψ via the attraction map. Thereafter the two states coalesce asymptotically. Thus the system as a whole (ψ and Ψ) starts near a pointer-basis state and ends near one. The probability for it to end near a given state is determined solely by the form of the active interactions which initiated the measurement. We can summarize the process as follows. In the initial phase, the local and global(factored) states, which begin very close together, are drawn apart. In the middle phase they undergo chaotic, (pseudo-)random motion. In the final phase capture of the local state by the global(factored) state occurs, very near a pointer-basis state which becomes the result of the measurement. We now consider random walk and capture phases in more detail.

7.1 Probability walk

In the middle phase of measurement the two states are typically not close together. The behavior of ψ is dominated by the chaotic map, although the attraction map does introduce a slight bias which breaks the symmetry of its distribution in the Hilbert space. It is important to consider the strengths of the chaotic map and of the attraction map. The first is determined by the ratio of the time scale Δt to the decohering interaction time scale. As long as these interactions are fast enough the 'chaotic' map will *actually be* in the strongly chaotic regime, characterized by a large Liapunov exponent and small correlation time. The second, the strength of the attraction map, is determined by an arbitrary parameter and by the degree of local coherence of the global state. This attraction will dominate only when the global state is almost totally locally coherent, near a pointer-basis state. Meanwhile the global state is affected by the global Hamiltonian, which tends to make its GDM diagonal in the pointer-basis, and by the attraction map, which should be weak as seen above. The resulting motion is a (pseudo-)random walk, because the local state is rapidly changing and filling in approximately the invariant distribution of the chaotic map. Here an important constraint must be observed[1]: During this motion, the *diagonal elements* of the GDM, $\hat{Q}$ (1), should remain approximately constant in time, *on average*.

To impose the constraint we will replace ψ by the distribution over the Hilbert space which the chaotic map generates. This is approximated by a symmetric sum of δ-functions at the pointer-basis states. Then we require that, for any $\hat{Q}$, the change in its diagonal components under the attraction map is zero, when

[1] I.e., the 'martingale' property; e.g., see Feller [4].

averaged over the distribution. This will be true if

$$\alpha(2+\alpha(1-z)) = constant \equiv \beta,\ 0 < \beta \leq 1\ . \tag{23}$$

The map becomes

$$|\Psi\rangle \rightarrow |\Psi'\rangle = \sqrt{1+\beta(1-z)}\mathcal{P}|\Psi\rangle + \sqrt{1-\beta z}(1-\mathcal{P})|\Psi\rangle\ . \tag{24}$$

It is instructive to see the effect on z of this process alone. Recall that z is the overlap between ψ and Ψ, ranging between 0 and 1. Under repeated application of the map,

$$z \rightarrow z' = z + \beta z(1-z) \rightarrow \cdots \rightarrow 1\ .$$

This map has fixed points at 0(unstable) and 1(stable), is the identity when $\beta = 0$, and forces $z \rightarrow 1$ most rapidly when $\beta = 1$. For now we will assume both the spread and bias of the invariant distribution is negligible. Failure for this to be strictly true will result in deviations from BP. Also, note that in the above prescription, the projector $\mathcal{P}$ (2) is used, rather than the density matrix $\hat{G}$, which is the more appropriate object because Trace$(\hat{G}) = 1$.

7.2 Capture

The final phase is the capture process, which gives meaning to the probabilities represented by the diagonal components of $\hat{Q}$. Normally, all fixed points of the combined attraction/chaotic map for ψ are unstable. But if $\hat{Q}$ is very nearly coherent and close to a pointer-basis state, this becomes a stable fixed point, an attractor. Soon ψ visits the neighborhood of this point and stays there, also holding in place Ψ, unless the active interactions pull it away too rapidly. The two states coalesce asymptotically for large time. The requirement that this picture should be valid, however, places some constraints on the local state maps. First, the attraction map is linear near the pointer-basis states (except see below), so the chaotic map must also be linear there. If the power is less than one, no capture will ever occur. If it is greater than one, capture will occur too soon and the probabilities will be violated. Therefore, as is implicit in the above, only the strength of the chaotic map is arbitrary but not its power-law behavior. Second, the attraction map strength on the local state must be at or near maximum in the following sense. The strength is defined by the constant γ, with $b^2 = 1-\gamma z$, which ranges between 0(weakest) and 1(strongest). For capture to occur γ must be at or near 1. This suggests that if the environment degrees of freedom are very numerous γ should approach one from below, yielding the greatest efficiency for measurement in 'macroscopic' experimental set-ups, as we would expect.

To see this, we consider the capture process in some detail. We assume for simplicity a two-level local system ($N=2$), and consider the effects of the chaotic and attraction maps on ψ, when both it and Ψ are near a pointer-basis state. We define $p_1 = 1-x$, $Q_{11} = 1-P$, with x and P, the probability-distances of the local and global states from the pointer-basis state, very small. The effect of the attraction map on x is

$$\begin{aligned} (\gamma < 1) \quad x \rightarrow x' \quad &\approx (1-\gamma)x\ , \\ (\gamma = 1) \qquad\qquad\ \ &\approx f(P)x\ , \end{aligned}$$

$$f(P) = (\sqrt{1-\gamma(1-P)}\frac{1-2P}{1-P} + \frac{P}{1-P})^2\ .$$

The effect of the chaotic map on x is

$$x' \rightarrow x'' \approx (1+\lambda^2)^2 x'\ ,\quad \lambda = \frac{\Delta t}{t_E} \gg 1 \ \text{ for chaos.}$$

We now let both maps act in sequence, and require that ψ be drawn even closer to the pointer-basis state. I.e., $x'' < x$ when $P < P^*$ and $x < x^*$, for some small P^* and x^*. If $\gamma = 1$, capture always occurs because the attraction map becomes quadratic as $P \to 0$ and overpowers the linear chaotic map. In this case $P^* \approx \lambda^{-4}$: For chaotic motion, this is a very small number, as it should be if large probability deviations, which are at least of order P^*, are to be avoided. If $\gamma < 1$, the attraction map is always linear and must compete with the chaotic map: Capture will not take place if E, the scale of decohering interactions, is large enough. For now we assume that $\gamma = 1$, so that capture takes place.

7.3 Measurement time

It is important to consider the average measurement time interval: how long a typical measurement requires. We can make an estimate based on the properties of random walks, in units of Δt. We first consider z to be undergoing an unbiased random walk:

$$z \to z \pm \beta z(1-z) \ , \ \pm = \text{random}. \tag{25}$$

Then we transform to a variable s with $\Delta s = \pm 1$, and use the standard result in Feller [4]. The average measurement time is

$$T_{meas} \approx (\ln[1 + (\Delta t/t_E)^2]^2)^2 \Delta t/\beta^2 \approx 16(\ln[\Delta t/t_E])^2 \Delta t/\beta^2 \ . \tag{26}$$

We can see what the minimum for this average measurement time will be by noting that the ratio $(\Delta t/t_E)^2$ is the strength of the map induced on the local state by the decohering interactions. This must be large enough for the map to be in the chaotic regime: $(\Delta t/t_E)^2 \geq \rho$, where $\rho \approx 8$. Hence the minimum average time for *any* measurement to occur is

$$T_{meas,min} = 4(\ln[1+\rho])^2 \Delta t/\beta^2 \ .$$

We can use this right away to constrain β, given some experimental input. If we observe a quantum measurement to occur during a time interval less than $T^\star$, then β must be greater than $\beta^\star$, where

$$\beta^\star = 2\ln[1+\rho]\sqrt{\Delta t/T^\star} \approx 5\sqrt{\Delta t/T^\star} \ .$$

Using $\Delta t \approx 10^{-43} s$, we consider a few possibilities.

$$\begin{aligned} T^\star &= 10^{-23}(\text{strong interactions}) \Rightarrow \beta^\star \approx 10^{-9} \ , \\ &= 10^{-13}(\text{electromagnetism}) \Rightarrow \beta^\star \approx 10^{-14} \ , \\ &= 10^{-1}(\text{human perception}) \Rightarrow \beta^\star \approx 10^{-20} \ . \end{aligned}$$

At present, there is no definitive evidence of any quantum measurements occurring faster than the human perception scale, because otherwise we would have *proof* of the need for the 'collapse of the wavefunction', ruling out 'no-collapse' models. Hence the only bound provided is $\beta > 10^{-20}$: Even this is not very firm.

The uncertainty relation(UR) for time-energy holds that the measurement time should be at least as long as the decoherence time: $T_{meas} E \geq \hbar$. This is true automatically:

$t_E < \Delta t$**:** In the chaotic regime, the measurement time is always much longer.

$t_E > \Delta t$**:** In the non-chaotic regime, the measurement is not produced by the model, but is instead deferred until a later time, when other degrees of freedom and/or interactions become involved.

Indeed, the only problem is that, in the chaotic regime where measurements actually occur, T_{meas} is too long. This cannot be helped because the model does not attempt to describe reality at time scales below Δt: That is the main reason why effective discrete-time maps are used. If an interaction crosses a cut *and* it exceeds $\Delta E \equiv \hbar/\Delta t$ sufficiently, it will cause a measurement, which will take longer than one might have expected. From an experimental point of view, however, this is not bad: It would be much more difficult to observe measurement times which were shorter than UR allows, than times which are longer.

To summarize, MOM results in measurements which have a minimum average duration. This could be used to constrain β, if suitable experiments verifying the existence of the collapse process were performed: Otherwise, we can still say $\beta > 10^{-20}$, based on the human perception time scale. In MOM, measurements occur only at decoherence energies above the energy scale ΔE, and then the measurement time exceeds the value prescribed by UR.

7.4 Probability deviations

The frequencies for outcomes of measurements in MOM deviate from BP due to at least two effects. First there is the finite size of the region for capture of the local state by the global state, and second there is the fact that the time-averaged distribution of the local state, as it undergoes chaotic motion, is not perfectly and symmetrically concentrated at the pointer-basis states. Here we will discuss these two effects in turn.

The effect of the finite size of the capture region can be seen by returning to the basic probability walk introduced in §1.3. In the one-dimensional case, let the diagonal element of the GDM be z, between 0 and 1. The usual BP is just $p \equiv z$, and one can check that the random walk keeps this constant, on average, at each step: This was the constraint used in §7.1 to ensure BP would be reproduced by the model(approximately). The *true* probability $q(z)$ obeys the boundary conditions

$$q(\delta) = 0, \;\; q(1-\delta) = 1, \;\; \delta = (\Delta t/t_E)^{-4} \ll 1,$$

and the self-consistency equation

$$q(z) = \frac{1}{2}[q(z_+) + q(z_-)] \;, \;\; z_\pm = z \pm \Delta z \;, \;\; \Delta z = \beta z(1-z). \tag{27}$$

This just says the value of q at the mid-point between z_+ and z_- is the average of the values at the two points. The solution should be a linear function of z, namely

$$q(z) = \frac{z-\delta}{1-2\delta} \;,$$

in which case the maximum deviation from BP, occurs at $z = \delta$ and $z = 1-\delta$, and has the value δ. The only problem with the above derivation is, that due to the finite size of β, the linearity cannot actually hold over a *finite* interval, when z is within about $\beta\delta$ of a capture region. The actual solution for q seems to be a series of steps, which tend toward a straight line, in the limit as $\beta \to 0$. Since β is small by assumption, we assume that the deviation from linearity for $q(z)$ is no worse than $\beta\delta$, and therefore our estimate is approximately correct. I.e. the deviation from BP due to the finite size of the capture region is no worse than the size of the region, $\delta = (\Delta t/t_E)^{-4} \ll 1$.

The second set of effects is due to the fact that the time-averaged distribution of the local state is not ideal. There are two sources of deviation: spread and bias.

Spread: The local states do not go precisely to some pointer-basis state after each application of the chaotic map. Their distribution has a spread about each of these, which is small because of the shift map. In §5, we find this spread to be about the same as the exponential factor above, $\exp -(\Delta t/t_E)^2 \ll 1$.

Bias: The complete chaotic map for the local state has no bias, in the sense that its distribution is symmetric under exchange of pointer-basis states. While most paths are not symetrically distributed, the average over all possible initial states for the path *is* symmetric. The only bias is introduced by the attraction of the GDM, which is *not* usually at a symmetric point(i.e. multiple of the identity). Thus in the two-state system, the GDM will push the distribution toward one of the two states, away from the other, at any given time. This effect is very small, because of the small spread of the distribution and the large mixing of the chaotic map. First consider the case, where the GDM is diagonal in the pointer basis. For any local state which is near a pointer state, the GDM has a negligible effect (unless it is inside the capture region!). It can only succesfully pull, from one half of the Hilbert space to the other, those local states which have comparable components of the two basis states. But the magnitude of the distribution away from these is suppressed by the factor $\exp -(\Delta t/t_E)^2$, according to §5. Hence this effect should be supressed by this factor. If the GDM is not diagonal, then it has a sizable effect on the tight distribution about each pointer-basis state, pulling it a considerable distance around the Hilbert space. However, before the distribution is sampled by the global state, another cycle must be completed, during which the chaotic map will mix the distribution beyond recognition, and return it to the vicinity of the pointer-basis states. If this mixing is strong enough ($\Delta t/t_E$ big enough), the bias will be eliminated, or replaced with a bias which is statistically unrelated to the density matrix. It is difficult to estimate this effect, and below we will assume the GDM is diagonal, due to the strong decohering interactions.

Now let us see what effect these deviations from ideality will have on the true probabilities $q(z)$. In the two-state case, the local state is distributed in the following way. With probability $(1-\delta f)/2$, it is within δ_0 of the state coresponding to $z=0$, while with probability $(1+\delta f)/2$, it is within δ_1 of the state corresponding to $z=1$. This alters the average value of z for the GDM after one cycle, or equivalently the true probability q.

$$q(t) \to \bar{q}(t+\Delta t) = q(t) + \overline{\delta q}(t),$$

where $\overline{\delta q} = 0$ in the idealized case, but here instead

$$\overline{\delta q} = \beta q(1-q)[\delta f(1-\delta_+) - \delta_-], \;\; \delta_+ = \delta_1 + \delta_0, \;\; \delta_- = \delta_1 - \delta_0.$$

The number of cycles that pass, on average, before the capture process results in a measurement, is about β^{-2}, and the effect is (mostly)cumulative, rather than difussion-like. Therefore the expected total drift should be

$$\begin{aligned} \overline{\delta q}_{total} &\approx q(1-q)[\frac{\delta f - \delta_-}{\beta}], \\ &\approx e^{-(\Delta t/t_E)^2}/\beta, \\ &\ll \delta = (\Delta t/t_E)^{-4}. \end{aligned} \tag{28}$$

The conclusion is, therefore, that the dominant factor in making the frequencies of measurement outcomes differ from BP is the finite size of the capture region δ. Thus the largest deviations from BP will be found whenever the decohering interactions, which actually cause a measurement, are not far above the threshold for the measurement to take place, i.e. just inside the chaotic regime.

II: Applications

> Before Zen, a tree is a tree and a mountain is a mountain. During Zen, a tree is not a tree and a mountain is not a mountain. After Zen, a tree is again a tree and a mountain is again a mountain. *Chuang-Tzu*

8 Standard Measurement Situations

In this section we try to connect the result of Part I to real measurements. First we discuss a simple detector, which registers the fact that its environment has reached a certain pre-selected state. Then the detector is generalized to select from among a number of possible states of a system of interest. Finally we consider the effect of kinetic terms in the local Hamiltonian, which lead to intermittent behaviour, i.e. jumping around from one pointer basis state to another.

8.1 A simple detector

In the simplest measurement scenario, we call the local degrees of freedom the detector, which monitors the environment for a certain state $|a_0\rangle$. The local and global states are

$$\begin{aligned} \text{local} \quad &: \quad |s\rangle = \sum_n s_n |n\rangle, \quad n = 0, 1 \\ \text{global} \quad &: \quad |\psi\rangle = \sum_{na} \psi_{na} |na\rangle, \quad n = 0, 1; \quad a = 1, 2, \ldots, A. \end{aligned}$$

The required Hamiltonian is the sum of three parts:

$$\begin{aligned} \hat{H} &= \hat{H}_{env} + \hat{H}_{dec} + \hat{H}_{meas} && (29) \\ \hat{H}_{env} &= \text{kinetic energy of environment} \\ &= \sum_{ab} K_{ab} |b\rangle\langle a| \otimes \sum_n |n\rangle\langle n| && (30) \\ \hat{H}_{dec} &= \text{decohering interactions} \\ &= \sum_{na} E_{na} |na\rangle\langle na| && (31) \\ \hat{H}_{meas} &= \text{active interactions} \\ &= B(|1a_1\rangle\langle 0a_0| + |0a_0\rangle\langle 1a_1|), \quad \langle a_0|a_1\rangle = 0. && (32) \end{aligned}$$

We first show that the effective local Hamiltonian can be neglected. It is calculated as above, by assuming no information about the environment, or equivalently tracing the Hamiltonian over the environment degrees of freedom.

$$\hat{H}_{local} = \sum_{mn} V_{mn} |n\rangle\langle m|$$

$$\begin{aligned} V_{mn} &= \frac{1}{A} \sum_a \langle ma|\hat{H}|na\rangle \\ &= \frac{\delta_{m,n}}{A} \sum_a (K_{aa} + E_{na}) \\ &= \delta_{m,n}(K + \frac{1}{A} \sum_a E_{na}) \\ &= \delta_{m,n} \times \text{constant, by assumption.} \end{aligned}$$

The only effect the Hamiltonian has on the local state is therefore the map induced by $\hat{H}_{dec}$, which is assumed to be in the strongly chaotic regime in order to effect a measurement. The presence of $\hat{H}_{meas}$ has a negligible impact on this evolution, provided B is small compared to the decoherence energy. To see this,

we calculate the decoherence function $\mathcal{Z}$ which generates the map. Proceeding as in §4.2, we find that it is the sum of two parts:

$$\begin{aligned}
\mathcal{Z} &= \mathcal{Z}_{dec} + \mathcal{Z}_{meas}, && (33)\\
\mathcal{Z}_{dec} &= \frac{1}{2}\left(\frac{E_{dec}\Delta t}{\hbar}\right)^2 \sum_n p_n(1-p_n), \;\; p_n = \|s_n\|^2, && (34)\\
&= \text{the usual term,} \\
\mathcal{Z}_{meas} &= \frac{1}{NA}\left(\frac{B\Delta t}{\hbar}\right)^2 (p_0^2 + (1-p_0)^2) \ll 1, && (35)\\
\mathcal{Z}_{meas}/\mathcal{Z}_{dec} &\sim \left(\frac{B}{E_{dec}}\right)^2 /(NA) \ll 1. && (36)
\end{aligned}$$

Note that it is actually the gradient, in the local Hilbert space, of $\mathcal{Z}$ which determines the map. Hence the fact that, at the pointer basis states, $\mathcal{Z}_{dec} = 0$ while $\mathcal{Z}_{meas} \neq 0$ is not important. So, the local state is governed by the usual chaotic map, which makes it jump pseudo-randomly between the pointer basis states $|n\rangle$.

Let us discuss the role of the different terms in the Hamiltonian as they affect the global state, and then go through the measurement itself from beginning to end. $\hat{H}_{dec}$ is a strong decohering term and will force the GDM,

$$\hat{Q} = \sum_a \langle a|(|\psi\rangle\langle\psi|)|a\rangle = \sum_{mna} \psi_{ma}\psi^*_{na}|m\rangle\langle n| \;,$$

to remain diagonal throughout the process. I.e. $\hat{Q} \to \sum_n p_n|n\rangle\langle n|$. $\hat{H}_{meas}$ will effect the measurement by moving the detector away from its set(initial) state when the environment is in the selected state $|a_0\rangle$. $\hat{H}_{env}$ has the effect of mixing up the environment states. This will make it possible for the measurement to require a finite amount of time before occurring, and for the outcome to persist for some finite period of time. Let us consider the evolution of the global state in the absence of the local one, and then see how this is affected by its presence. The state begins at

$$|\psi_0\rangle = |0\rangle \otimes \sum_a C_a|a\rangle, \;\; \|C_{a_0}\|^2 = 0.$$

As a result of $\hat{H}_{env}$, the state eventually moves to

$$|\psi'_0\rangle = |0\rangle \otimes \sum_a C'_a|a\rangle, \;\; \|C'_{a_0}\|^2 > 0.$$

Now we need to anticipate a result of §8.3: The detector decays toward the triggered state via an exponential(approximately) given by $k = e^{-\lambda t}$, where $\lambda \sim B^2/(\hbar E_{dec})$. The state becomes

$$|\psi(t)\rangle = k|0\rangle \otimes |A_0(t)\rangle + \sqrt{1-k^2}|1\rangle \otimes |A_1(t)\rangle,$$

where the environment states are complicated. Eventually(asymptotically as $t \to \infty$), the state reaches the triggered state: $|\psi_\infty\rangle = |1\rangle \otimes |A_\infty\rangle$. Thus the detector will be triggered, if the environment state remains near the selected state $|a_0\rangle$ forever. More realistically, and for finite times, the probability of triggering will not equal one, and the quantum description of the detector will remain a mixture of the two states. In practice, either the detector is triggered or it is not, with a probability which depends on the experimental arrangement, i.e. the initial state of the environment and $\hat{H}_{env}$.

Now let us see how MOM affects the evolution of the detector. At the beginning, the local and global states agree on the detector state, and the environment is not yet ready to trigger the detector:

$$|s\rangle = |0\rangle, \;\; |\psi\rangle = |0\rangle \otimes \sum_a C_a|a\rangle, \;\; \|C_{a_0}\|^2 = 0.$$

A period of time follows during which nothing happens to the detector, because $\|C_{a_0}\|^2$ is too small. Although the active interactions try to move p_0 away from 1, the attraction to the local state keeps it there: A threshold must be exceeded by $\|C_{a_0}\|^2$ at some point, or the detector will not be triggered.

To see this, consider the combined effect of the active interaction(decay) and the attraction to the local state on the GDM. First allow the attraction map to operate, recalling that we expect $\beta \sim 1/A \ll 1$:

$$p_0 \to p_0' = p_0 + \beta p_0(1 - p_0).$$

Then allow the decay:

$$p_0' \to p_0'' = p_0' e^{-\epsilon}, \;\; \epsilon = \|C_{a_0}\|^2 B^2 \Delta t/(\hbar E_{dec}).$$

We want to see the total effect on the difference $\delta = 1 - p_0$. Neglecting small terms, and setting $\delta = \delta''$, we reach after some algebra the condition $\delta = \epsilon/\beta \equiv \delta^*$. For $\delta < \delta^*$, δ grows, while for $\delta > \delta^*$, it decreases: It is an attracting fixed point. Therefore $p_0 \to 1 - \delta^*$ as long as the local state remains stuck at $|0\rangle$. The question then arises, whether δ^* is small enough to be still in the capture region. In Part I(§7.2) we found this region to be bounded by $p_0 > 1 - p^*$, $\;p^* \sim (\Delta E/E_{dec})^4$, where E_{dec} is the strength of $\hat{H}_{dec}$, and $\Delta E = \hbar/\Delta t$. Now, if $\delta^* < p^*$, the local state will remain trapped and the decay will never occur, while if $\delta^* > p^*$, the local state will soon leave $|0\rangle$ and undergo chaotic motion due to the effect of $\hat{H}_{dec}$. In this case, the probability p_0 will execute a pseudo-random walk, as described in Part I. The condition on $\|C_{a_0}\|^2$ for the local state to be released is

$$\|C_{a_0}\|^2 > \beta \frac{\Delta E^5}{B^2 E_{dec}^3} \equiv P_{crit}. \tag{37}$$

There are two possibilities. Either $P_{crit} < 1$ and release *may* occur, or $P_{crit} > 1$ and release *cannot* take place. This becomes a condition on B for the detector to be triggered at all:

$$B^2 > \beta (\Delta E)^2 (\Delta E/E_{dec})^3. \tag{38}$$

Note that we are assuming the ratio $\Delta E/E_{dec}$ is small, or the local state dynamical map would not be chaotic, and the random walk would not occur. Hence this condition could to be satisfied.

Once the local state has been released from $|0\rangle$ it will undergo a chaotic dynamics, jumping between the two basis states, $|0\rangle$ and $|1\rangle$. This will cause the global state to undergo the pseudo-random probability walk treated in Part I, except that it will also be moving toward $p_1 = 1$, with a slow exponential decay. There will be two competing effects: the slow decay and the fast tendency to reach a pointer basis state. To see how these work together, it is crucial to note that the probabilities $p_n(t)$ are *not* affected by the random walk. Although the detector tends to reach a pointer basis state quickly, it does so strictly according to the current probabilities $p_n(t)$: No 'Zeno' effect occurs, which would change the expected time of triggering, because of the exponential form of the decay. The decay has that form because of the environment kinetic terms, which effectively dampen the motion. Once we assume the exponential decay form, it is easy to see that the random walk decouples from it. Let $p_0 = p$ at some time t. According to MOM, during time Δt the average value of p_0 will evolve as

$$\begin{aligned} p \quad \to p' \quad &= \frac{1}{2}(p + \delta p) + \frac{1}{2}(p - \delta p) \\ \to p'' \quad &= \frac{1}{2}e^{-\epsilon}(p + \delta p) + \frac{1}{2}e^{-\epsilon}(p - \delta p) \\ &= p e^{-\epsilon}, \end{aligned}$$

which is the same as without the random walk. Note two important facts. First, it is only because of the *exponential* form of the decay that this works. Second, there will be small deviations from the standard

probability evolution due to two factors: the (non-exponential) quadratic nature of all such decays for very short times, and the finite size of p^*. The quadratic effects should be small, unless the decay is still quadratic for $\delta \sim \delta^*$, in which case the above analysis is invalid: We assume this is not the case. The other deviations should be only of order p^*, so when the detector does trigger, it should do so after being exposed to the selected environment state $|a_0\rangle$ for the usual length of time. The triggering does not last forever, however, because the environment will eventually return to the vicinity of the state $|a_1\rangle$, and the detector will then start moving back toward $|0\rangle$, in a manner analogous to the initial departure from that state. This presents a picture of the detector switching at unpredictable intervals between the two states. However, we should take into account the fact that the environment has a very large number of states($A \to \infty$). This means that the recurrence time for the state $|1a_1\rangle$ is effectively infinite, so that the result of the detector triggering will be permanent. It also means that the initial waiting period for $\|C_{a_0}\|^2$ to grow above P_{crit} is also arbitrary. The final picture is that the detector sits for an indeterminate length of time, waiting to be triggered, then is triggered according to an exponential decay law during a relatively short period, and then rests in the triggered state indefinitely.

8.2 Multi-state detector/selector

In this measurement scenario, the local degree of freedom is still the detector, but in addition to the environment there is an 'object' system from which a state will be selected. The particular state is a member of the pointer basis and is not known in advance, but it will be chosen according to the usual probability rule. The local and global states are

$$\begin{aligned} \text{local} \quad &: \quad |s\rangle = \sum_n s_n |n\rangle, \\ \text{global} \quad &: \quad |\psi\rangle = \sum_{nia} \psi_{nia} |nia\rangle, \end{aligned}$$

$$n = 0, 1, \ldots, N; \;\; i = 1, \ldots, N; \;\; a = 1, 2, \ldots, A.$$

The required Hamiltonian is still the sum of three parts:

$$\hat{H} = \hat{H}_{env} + \hat{H}_{dec} + \hat{H}_{meas} \tag{39}$$

$$\hat{H}_{env} = \sum_{ab} K_{ab} |b\rangle\langle a| \otimes \sum_{ni} |ni\rangle\langle ni| \tag{40}$$

$$\hat{H}_{dec} = \sum_{nia} E_{nia} |nia\rangle\langle nia| \tag{41}$$

$$\hat{H}_{meas} = B \sum_i (|iia_i\rangle\langle 0ia_0| + |0ia_0\rangle\langle iia_i|), \;\; \langle a_0|a_i\rangle = 0. \tag{42}$$

As before, the effective local Hamiltonian can be neglected, as well as the self-energy of the object system. The story of this measurement process is similar to that above. The global state begins at

$$|\psi_0\rangle = |0\rangle \otimes \sum_i D_i |i\rangle \otimes \sum_a C_a |a\rangle, \;\; \|C_{a_0}\|^2 = 0,$$

and would, in the absence of the local state, evolve asymptotically to

$$|\psi_\infty\rangle = \sum_i D_i |ii\rangle \otimes |A_{i,\infty}\rangle.$$

Once the local state has been released from $|0\rangle$ it will undergo a chaotic dynamics, jumping between the basis states, $|0\rangle$ and $|i\rangle$. Finally the detector, object and environment will reach a semi-stable state $|\overline{n}\rangle \otimes |A_{\bar{\imath}}(t)\rangle$, with probability given by $\|D_{\bar{\imath}}\|^2$, as expected.

8.3 Intermittency

In the idealized version of measurements used above, the result of a measurement is permanent. However, actual measurements do not last forever. Sooner or later, the outcome of any measurement will fade away. This is due to the presence of kinetic terms in the effective Hamiltonian(self-energy) which are not diagonal in the pointer basis. Thus, after the position of a particle is measured, the wavefunction of the particle in space will spread out: A new measurement of the position will probably yield a different value. In terms of MOM, if the kinetic terms are big enough the system will show intermittent behavior: It will jump randomly amongst the pointer basis states at random intervals. However, if these kinetic terms are very small, they will not be effective and the system will become stuck permanently at one of these states.

In the simplest case we consider a two-state system with its environment:

$$\begin{aligned} \text{local} &: |s\rangle = u|u\rangle + v|v\rangle \\ \text{global} &: |\psi\rangle = \sum_a (u_a|ua\rangle + v_a|va\rangle), \quad a = 1, 2, \ldots, A. \end{aligned}$$

The required Hamiltonian contains a kinetic(self-energy) part and a decohering part:

$$\hat{H} = \hat{K} + \hat{D} \tag{43}$$

$$\hat{K} = \omega(|u\rangle\langle v| + |v\rangle\langle u|) \otimes \sum_a |a\rangle\langle a| \tag{44}$$

$$\hat{D} = \sum_a (E_a|ua\rangle\langle ua| + F_a|va\rangle\langle va|). \tag{45}$$

To see the effect of these two terms, we consider the GDM, $\hat{Q}$, and change over to the Riemann variables (x, y, z):

$$\hat{Q} = \begin{bmatrix} p & qe^{i\theta} \\ qe^{-i\theta} & 1-p \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(1+z) & x - iy \\ x + iy & \frac{1}{2}(1-z) \end{bmatrix}. \tag{46}$$

Now we write differential equations for (x, y, z). The kinetic term is easily handled, but the decohering interactions must be approximated. Their effect is to exponentially decay the off-diagonal parts of $\hat{Q}$: This is represented by terms proportional to λ, which represents the strength of these decohering interactions. Defining $k \equiv -2\omega$, the evolution is

$$\frac{dx}{dt} = -\lambda x \ , \quad \frac{dz}{dt} = -ky \ , \quad \frac{dy}{dt} = kz - \lambda y.$$

We let $x = 0$ for simplicity. This is a linear system and can be solved by finding the eigenvalues and eigenvectors of the time evolution operator. We are interested in the overdamped case: $R \equiv \lambda/(2k) \gg 1$. The eigenvalues are

$$\begin{aligned} \alpha_+ &= -k(R - \sqrt{R^2 - 1}) \approx -k^2/\lambda \ll k \\ \alpha_- &= -k(R + \sqrt{R^2 - 1}) \approx -\lambda. \end{aligned}$$

The eigenvectors are approximately

$$\begin{aligned} \vec{r}_+ &= \begin{pmatrix} z \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ k/\lambda \end{pmatrix} \approx \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \vec{r}_- &= \begin{pmatrix} k/\lambda \\ 1 \end{pmatrix} \approx \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \end{aligned}$$

Apparently, the net result is that the components (x, y) decay very rapidly, almost exactly as they would without the kinetic term, while the component z decays very slowly. The relationship between the time scales is

$$T_{decay}T_{decohere} = T^2_{kinetic}, \tag{47}$$

$$\begin{aligned} T_{decay} &= \lambda/k^2 : \text{z} - \text{decay time}, \\ T_{decohere} &= 1/\lambda : \text{decoherence or (x, y)} - \text{decay time}, \\ T_{kinetic} &= 1/k : \text{natural spreading time}. \end{aligned}$$

Now we want to consider the effect of the kinetic term on the random walk of the z-component of the GDM, and on the chaotic map of the local state. As long as $k \ll \Delta E$, the chaotic map will be unaffected, and the local state will behave overall as usual. The random walk for z will be as in §7.1, except that $|u\rangle$ and $|v\rangle$ are *not* in general absorbing states. By a similar analysis we can find the condition for the system to be able to escape $|u\rangle$ or $|v\rangle$, in case both the local and global states find themselves there. The kinetic terms must be strong enough:

$$k > k_{min} = \beta^{1/2}\Delta E(\frac{\Delta E}{\lambda})^{3/2}. \tag{48}$$

With our usual assumptions ($\beta \ll 1$ *and* $\lambda \gg \Delta E$) this is possible, since $k_{min} \ll \Delta E$. Assuming this is the case, z executes a random walk in the interval between $\pm(1 - \delta^*)$, where $\delta^* = \Delta t/(\beta T_{decay})$. This is the combination of the basic random walk due to the attraction to the local state, and the decay which makes $z \to 0$. We can see what the overall pattern of behavior will be by noting that the motion is very similar to a *random walk with reflecting barriers*[4]. Near the center, for $z \approx 0$, the decay is unimportant and the random walk is unbiased. At the edges, for $z = \pm(1 - \delta^*)$, z either stays unchanged or moves one step toward the center, with both probabilities equal to $1/2$. In between, there is a smooth transition from one type of behavior to the other. Assuming the analogy is valid, we can use Feller's results. Let us start z at an arbitrary state, and consider the distribution of values it will have after some large number of time steps. For an unbiased random walk with reflecting barriers and steps of the same size everywhere, this tends toward a uniform distribution. In our case the steps are not of the same size, rather they are roughly $\Delta z \approx \beta(1 - z^2)$. This means the distribution is concentrated where the steps are smallest, where $1 - z^2$ is small, but diminishes to zero near $z = \pm 1$. The effect of the z-decay, because it introduces a bias toward the center, will be to soften this distribution somewhat. If this is not overwhelming, we should see the following behavior. The system (global z) spends almost all of its time near the states $z = \pm 1$, with ocassional departures into the center: These last only for a short time, about $\Delta t/\beta^2$. If we start with the system at, e.g. $z = 1$, and ask what is the probability for the system to be there again at a later time, the answer is very nearly the standard exponential decay towards $1/2$, which we expected in the absence of the local state introduced by MOM. Again, this is because the exponential decay 'commutes' with the random walk. Note that in the standard interpretation of the behavior of the global state alone, without the added elements of MOM, this answer corresponds to a different question, namely , if we measure the system to be at $z = 1$, what is the probability that a subsequent measurement will give the same result? Within MOM description of the situation, no intervention by an outside observer is needed to explain the odd behavior of the system: It behaves this way already without measuring apparatus at hand. There are several experiments which exhibit this kind of intermittent behavior [7, 8]. The standard explanation for the behavior is that it is *as if* the state of the system, along the pointer basis, were measured periodically, resulting in an exponential decay law via the Zeno effect. The actual exponential decay rate depends on the frequency of these measurements, which is taken to be given by the decoherence time [9]. In MOM, there are no special assumptions needed to obtain the intermittency. I.e. this follows from the same general model which explains normal measurements. (See also §13).

9 Non-standard Situations

In this section we consider the possible effect of applying MOM to certain situations which are not like those of §8, but may correspond to observable behavior. In each case, we assume the domain inside the cut contains only a specific degree of freedom, which interacts with its environment in a specified way. Then we calculate the decoherence function $\mathcal{Z}$, from which we can anticipate qualitative aspects of the evolution of the combined local and global system.

9.1 Spin observer w. spin-vector interactions

One may take the local degrees of freedom to be the spin of some system, a field or particle, which interacts via a spin-vector dot product coupling with the environment. This environment can be a set of particles with spin, a vector or tensor field, or any system which allows a representation of a vector operator. We begin with the simplest case:

$$\text{local} : |s\rangle = \sum_m s_m |m\rangle, \;\; \text{global} : |\psi\rangle = \sum_{mm_e} \psi_{mm_e} |mm_e\rangle,$$

$$m = -j, -j+1, \ldots, +j; \;\; m_e = -j_e, -j_e+1, \ldots, +j_e.$$

The Hamiltonian is a spin-spin interaction with strength E:

$$\hat{H} = (E/\hbar^2)\vec{J}\cdot\vec{J}_e \; . \tag{49}$$

The effective Hamiltonian for the local state is zero, and the decoherence generating function is

$$\begin{aligned} \mathcal{Z} &= (1/3)(\frac{E\Delta t}{\hbar})^2 j_e(j_e+1)(\langle s|\vec{J}^2|s\rangle - \|\langle s|\vec{J}|s\rangle\|^2)/\hbar^2 \\ &= (1/3)(\frac{E\Delta t}{\hbar})^2 j_e(j_e+1)(j(j+1) - \|\vec{J}\|^2/\hbar^2) \; , \end{aligned} \tag{50}$$

where $\|\vec{J}\|^2 \equiv \|\langle s|\vec{J}|s\rangle\|^2$. Much can be determined from examination of this function. First note that the effective strength of the interaction is actually $E_{eff} = Ej_e$, which can be much greater than E. Now let us consider some simple examples. For $j = 0$ there is *no* local degree of freedom. For $j = 1/2$, $\mathcal{Z}$ is constant and there is no map induced on the local state: A spin-1/2 system local state is immune to the decoherence effects. As we will see below, this is because it already has its spin maximally aligned along some direction. Now let j be arbitrary, but restrict the state by $|s\rangle = |m\rangle$. Then $\|\vec{J}\|^2 = m^2$, and $\mathcal{Z}$ has a simple dependence on m. Assuming j to be an integer, we calculate the ratio R of $\mathcal{Z}$ for $m = 0$ to that for $m = j$: $R = j + 1$. This means that the gradient of $\mathcal{Z}$ becomes greater as j is increased. As we see below, this leads toward more classical behavior, for a given value of E.

Next we want to find the fixed points of the mapping induced on the local state. This map is proportional to the gradient of $\mathcal{Z}$, so these are its maxima and minima.

maxima: These are the points with $\|\vec{J}\|^2$ as small as possible, which are linear combinations of the states $|m\rangle$, with amplitudes of equal norm. These states are much like plane-waves or standing waves in mechanics. For very small values of E_{eff}, the shift map can be ignored. The map slowly pushes the local state toward one of these states, the closest one to the initial state. This will pull the global state also toward this kind of state, or perhaps toward an equal-probability mixture of these states. Note that this is decidedly *non-classical* behavior, because classical spins always have maximal value of $\|\vec{J}\|^2$.

minima: These are the points with $\|\vec{J}\|^2$ as large as possible, namely all states where $\langle s|\vec{J}|s\rangle = \hbar j\hat{e}$, where $\hat{e}$ is a unit vector in any direction. For large values of E_{eff}, the local state will be in the chaotic regime, and will be driven at the end of each cycle Δt to one of these states. As a result, the global state is always being pulled toward a *classical* state, i.e. one with definite spin direction.

Now we consider the gradient of $\mathcal{Z}$, which governs the local-state behavior as follows. If E_{eff} is small, the induced map is given just by the discrete map, and the local state tends toward the maxima of $\mathcal{Z}$, where the gradient is zero. If E_{eff} is large, the discrete map is chaotic, and the shift map becomes dominant. Recall that this map is the result of evolving the state $|s\rangle$ for time Δt under the equation

$$d|s\rangle/dt = -|\vec{\nabla}_\perp \mathcal{Z}\rangle.$$

Therefore the final state after each cycle will tend to be near some minimum of the gradient, which is given by

$$|\vec{\nabla}_\perp \mathcal{Z}\rangle = (k/\hbar^2)\hat{G}|s\rangle,$$

$$\hat{G} = -(1 - |s\rangle\langle s|)\langle s|\vec{J}|s\rangle \cdot \vec{J}\ , \quad k = (2/3)(\frac{E\Delta t}{\hbar})^2 j_e(j_e+1). \tag{51}$$

This equals zero only at the points $s_m = \delta_{m\bar{m}}$, $\bar{m} = -j, -j+1, \ldots, +j$. Defining ω and p_m:

$$\langle s|\vec{J}|s\rangle \equiv \hbar\omega\hat{z}, \quad p_m \equiv \|s_m\|^2,$$

the shift map is governed by

$$dp_m/dt = k\omega(m-\omega)p_m\ , \tag{52}$$

which in particular implies

$$dp_j/dt \geq 0\ , = 0 \quad \text{iff} \quad \omega = j : \text{classical state.}$$

This supports the statements made above concerning the qualitative behavior of the local state.

Now let us extend the local system and environment by allowing different values for the spin magnitudes:

$$\text{local} : |s\rangle = \sum_{jm} s_{jm}|jm\rangle, \ \ \text{global} : |\psi\rangle = \sum_{jmj_em_e} \psi_{jmj_em_e}|jmj_em_e\rangle,$$

$$j = 0, 1, 2, \ldots, J; \ \ j_e = 0, 1, 2, \ldots, J_e.$$

Up to numerical factors of order one, $\mathcal{Z}$ does not change, except that j_e is replaced by J_e. The gradient changes because of the expansion of the local Hilbert space, such that the new $\hat{G}$ is given by

$$\hat{G} = -(1 - |s\rangle\langle s|)(2\vec{J} - \langle s|\vec{J}|s\rangle) \cdot \vec{J}\,. \tag{53}$$

The result, for large E_{eff}, is that the local state, in addition to tending toward states with maximal spin projection along some axis as before, now also tends toward states with small values of j. In particular, $j = 0$ is favored above all others. To see this, define $p_j \equiv \sum_{m=-j}^{+j} \|s_{jm}\|^2$. The shift map is such that

$$dp_j/dt = 0 \text{ when } \ p_j = \delta_{j\bar{j}}\ , \ \ \bar{j} = 0, 1, 2, \ldots, J\ ,$$

and

$$dp_0/dt \geq 0\ , = 0 \ \text{ iff } \ p_0 = 0 \text{ or } 1\ .$$

Thus the local state is drawn inexorably toward $j = 0$. The only escape is the possibility that this tendency will be ameliorated by the attraction to the global state. Indeed, it is not likely that the global state can actually go to and *stay* at $j = 0$, because of other interactions which we have ignored. For example, if the

local system spin is associated with a particle, this particle's position(and spin) will interact with other particles' positions(and spins): This will likely keep the spin away from zero. Absent a detailed analysis, we can guess that the actual behavior of the spin system would be that it will tend to maintain a relatively low value(thermally determined?) for its average value of j, and that its spin will be maximally aligned along some direction. This direction is arbitrary, and would change due to collisions and other interactions.

In summary, the qualitative behavior of the local spin is as follows. If the system is $j = 0$ or $1/2$, then it is unaffected. For any other system with fixed j, the dynamics depends on whether E_{eff} is small or large. If it is small, then the state is driven toward a uniform state. If it is large, it acts to force the spin to have maximal projection along some spatial axis. The actual direction is arbitrary and will depend on the initial conditions. Note that this is the behaviour of classical spinning systems. If j is not fixed, but can take any values, then the dynamics has the same effect, but it also forces the length of the spin to take the minimum value possible. This will be either $j = 0$ or $1/2$, but other interactions(which were neglected above) may force this value to rise. Note that there should be no intermittency effect, as such, for spin observables, because the relevant kinetic terms are absent from the Hamiltonian.

9.2 Position observer w. Coulomb-type interaction

One may define the local degree of freedom to be the set of positions of some particles, which interact with other particles or a matter-density field through a Coulomb-type gravitational term. The simplest case is where the local system and its environment consist of just one particle each. For convenience, let us use a discretized version of three-dimensional space, with a finite number N of sites, each of small size R_1. If the size of the universe is R, we have approximately $N = (R/R_1)^3$. The states are

$$\text{local} : |\psi\rangle = \sum_a \psi_a |a\rangle, \;\; \text{global} : |\Psi\rangle = \sum_{au} \Psi_{au} |au\rangle,$$

$$a, u = 1, 2, 3, \ldots, N.$$

The decoherence function depends only on a Coulomb-type interaction representing gravity. If D_{ab} is the positive distance between two sites($D_{aa} \equiv R_1$), then the potential is $V_{ab} = k/D_{ab}$, where $k = Gmm'$, G is Newton's constant, m is the mass of the local-state particle, and m' is the mass of the environment particle. Then the (interaction) Hamiltonian is

$$\hat{H}^{int} = \sum_{au} |au\rangle\langle au| V_{au} \; . \tag{54}$$

The contribution from this interaction to the effective Hamiltonian for the local state is a constant:

$$\hat{H}^{int}_{eff} = -M \sum_a |a\rangle\langle a| \; , \;\; M = (k/N) \sum_b D^{-1}_{ab} : \text{independent of a.}$$

The decoherence generating function $\mathcal{Z}$ is

$$\mathcal{Z} = (\Delta t/\hbar)^2 (\sum_a p_a K - \sum_{ab} p_a p_b K_{ab}) \; , \tag{55}$$

where $p_a \equiv \|\psi_a\|^2$, and

$$K = (k^2/N) \sum_b D^{-2}_{ab} : \text{independent of a,}$$

$$K_{ab} = (k^2/N) \sum_c D^{-1}_{ac} D^{-1}_{bc} : \text{dependent only on } D_{ab} \; .$$

Before proceeding further, it is instructive to look at the simplest examples.

localized state: If $\psi_a = \delta_{ab}$, then $\mathcal{Z} = 0$. I.e. the lowest possible value for $\mathcal{Z}$ is realized by any wavefunction which is completely localized. In fact, these are the *only* states for which $\mathcal{Z} = 0$, barring accidental degeneracies due to space curvature. In the strongly chaotic regime, when the local-state map is dominated by the shift map, it nearly always returns to one of these states after each cycle Δt. The local state will jump around between these localized states, and the global state will perform a probability walk as in §7.1. Thus we should see a tendency for the particle's position to become well-defined, which will be partially offset by the effect of kinetic energy terms.

two-site state: If $\psi_a = \sqrt{p}\delta_{ab} + \sqrt{1-p}\delta_{ac}$, for $b \neq c$, then $\mathcal{Z} \propto p(1-p)$. This is just like the generic case of §7.1. When the strength of the interaction is weak, the local state moves slowly toward uniformly-distributed states(plane-waves and standing-waves). When the interaction is strong enough, it jumps randomly between the two localized states. Taking the kinetic energy terms into account, the particle will either: a)not be affected at all, for extremely weak interaction, b)become a good-momentum state, for moderately weak interaction, or c)become intermittently localized, for strong interaction.

To obtain the general form of $\mathcal{Z}$ and judge the strength of the interaction, we replace the finite sums with continuum integrals and evaluate them using a crude regularization procedure, where all space integrals are cut off at the distance R. We obtain

$$\begin{aligned} M &= (3/2)k/R, \\ K &= 3(k/R)^2 = (4/3)M^2, \\ K_{ab} &= (3/2)(k/R)^2(2 - D_{ab}/R) = (2/3)M^2(2 - D_{ab}/R). \end{aligned}$$

From this it follows that

$$\mathcal{Z} = (2/3)(\frac{M\Delta t}{\hbar})^2 \bar{D}/R \ , \quad \bar{D} = \sum_{ab} p_a p_b D_{ab}, \tag{56}$$

where $\bar{D}$ will be referred to as the *spread*. The need for regularization makes it difficult to tell whether this is large or small, compared to one, in any particular case. To solve this problem, let us assume that all constants appearing are to be replaced by Planck quantities, except for the particle mass m, and try to justify this replacement *a posteriori*. I.e., let

$$\mathcal{Z} \to (m/M_p)^2 \bar{D}/R_p \ , \tag{57}$$

where(speed of light $c = 1$)

$$\begin{aligned} M_p &= 1.2 \times 10^{28} eV : \textit{Planck mass}, \\ R_p &= 1.6 \times 10^{-36} m : \textit{Planck length}. \end{aligned}$$

This will be true if $M^2/R = m^2/R_p$, which is equivalent to $1 = (Gm'/R)\sqrt{R_p/R}$, or $m' = M_p\sqrt{N}$.

Now we consider the following alternate situation: Let the environment consist, instead, of N' particles of mass m', each interacting with the local system particle in the same way as above. Then $\mathcal{Z} \to N'\mathcal{Z}$, which is the same as before, except that $m' \to m'\sqrt{N'}$. We reach the desired result by letting $m' \equiv M_p$, and $N' \equiv N$: The particle of interest is interacting with a *sea* of particles, each with mass M_p, of which there is one for each Planck-sized volume in the universe. This may be hard to accept at face value, but we can imagine that these are somehow *virtual* particles, associated with some deep aspect of sub-Planck-scale spacetime, and otherwise invisible to most probes. Perhaps they represent the vacuum energy of some field.

At any rate, this development leads to interesting results. We can ask ourselves, at what length scale does the localization of the particle become dominant. I.e., for a given mass m, what spread D will give $\mathcal{Z} \approx 1$. Putting in the appropriate constants, we can evaluate D for any mass:

electron: $m = 5 \times 10^5 eV \rightarrow D = 10^9 m$. This is certainly beyond any present experimental bound.

proton: $m = 1GeV \rightarrow D = 200m$. This is still large, but perhaps small enough to be interesting.

heavy ion/atom: $m = 100GeV \rightarrow D = 2cm$. This could be experimentally accessible.

bio-molecule: $m = 10^6 GeV \rightarrow D = 2 \times 10^{-10} m = 2$Å. This is very small, but might be difficult to rule out because of the difficulty in handling such large molecules.

Planck-mass $m = M_p \rightarrow D = R_p$. Such a heavy particle would remain localized at or below the Planck-length. If this is an elementary particle, neither MOM nor any established physics can describe it: It makes no sense for a particle to have such a large mass. A composite object could easily have such a large mass, and then, *if there were a local state associated with its center of mass*, its position would practically become a *classical* object.

To summarize, the effect of an associated local state on the dynamics of a particle depends on the relation of the particle's spread to the characteristic distance D, which is inversely proportional to its mass. The relevant quantity is the spread of the global state, because the local state is strongly attracted to it, and will share its overall shape, i.e. most amplitude ratios. If the spread is extremely small, there is no effect and the particle behaves normally. If the spread is small but approaching D, the particle state tends towards approximate plane-waves of size less than D. Finally, if the spread is large compared to D, the particle undergoes spontaneous intermittent localization, as the kinetic energy of the particle competes with the localization effect. On a given distance scale, it is possible for light particles to be unaffected by the local state, but for heavy particles to have their non-classical spread severely restricted. The magnitude of the kinetic terms in the Hamiltonian is greater for particles with small mass and lesser for heavier ones. Hence the intermittency would be very slow, and possibly stop altogether, for very massive particles.

9.3 Bose-Einstein and Fermi-Dirac statistics

Here we want to consider the evolution of a number of states, all in the same Hilbert space, when the Hamiltonian is zero. Only the universal attraction between states will be operating. The simplest case is when there are two states, $|U\rangle$ and $|V\rangle$. They will coalesce asymptotically to the linear combination $|t \rightarrow \infty\rangle \propto (|U\rangle + |V\rangle)$, and this is monitored by following the evolution of the overlap probability, $z \equiv \|\langle U|V\rangle\|^2$:

$$\begin{aligned} |U\rangle \rightarrow |U'\rangle &= (A\hat{P}_V + B(1-\hat{P}_V))|U\rangle \,, \\ |V\rangle \rightarrow |V'\rangle &= (A\hat{P}_U + B(1-\hat{P}_U))|V\rangle \,, \\ z \rightarrow z' &= R^2 z \,, \end{aligned}$$

where

$$\hat{P}_U = |U\rangle\langle U| \,, \quad \hat{P}_V = |V\rangle\langle V| : \text{projection operators,}$$

$$A = \sqrt{1+\beta(1-z)} \,, \quad B = \sqrt{1-\beta z} \,,$$

$$R = (A-B)^2 z + 2B(A-B) + B^2 \,.$$

First we note that, if the two states are identical or orthogonal, they remain so. Now we want to look at nearby cases:

$z \simeq 0$**:** The states are nearly orthogonal. $z' = (1 + \lambda(\beta))z$,

$$\begin{aligned} \lambda(\beta) &= 4\sqrt{1+\beta}(\sqrt{1+\beta}-1) > 0 \text{ , for } \beta > 0 \text{ ,} \\ &\stackrel{\beta\to 0}{\longrightarrow} 0 \text{ ,} \\ &\stackrel{\beta\to 1}{\longrightarrow} 4\sqrt{2}(\sqrt{2}-1) \text{ ,} \\ &\stackrel{\beta\to\infty}{\longrightarrow} \infty \text{ .} \end{aligned}$$

The larger the value of β, the faster the two states begin to approach each other.

$z \simeq 1$**:** The states are nearly identical. Let $x \equiv 1 - z$, then $x' = (1 + \lambda(-\beta))x$,

$$\begin{aligned} \lambda(-\beta) &< 0, \text{ for } \beta > 0, \\ &\stackrel{\beta\to 0}{\longrightarrow} 0, \\ &\stackrel{\beta\to 1}{\longrightarrow} 0, \\ &= -1, \text{ for } \beta = 3/4. \end{aligned}$$

The choice $\beta = 3/4$ yields the fastest approach to $z = 1$, analogous to critical damping of an oscillator. Apparently for larger β, the attraction is too large, and the states 'overshoot' past each other(underdamping). The values $\beta = 1$(zero damping), and $\beta = 0$(infinite damping), yield the slowest possible approach, for small x. It is not clear whether this feature of the attraction is a real effect, or an artifact of the discrete-time 'approximation.'

It is noteworthy, that the evolution of z for nearly-orthogonal states is the same as that of x for nearly-identical states, except with $\beta \to -\beta$. This suggests the following. If we were to allow negative values($\beta = -1$) for some Hilbert spaces(domains), while retaining a positive value($\beta = +1$) for others, the difference in behavior would be very much like the difference between *fermions* and *bosons*. For positive β(bosons), the states are attracted to each other. They will begin to coalesce quickly, but critically slow down as they approach a common state(for $\beta = 1$). For negative β(fermions), the states repulse each other. They will be quickly pushed apart, toward mutually orthogonal states, but will also critically slow down at the end(again, for $\beta = -1$). In this case, however, it is clear that if there are more states than basis states, they will be frustrated, and will become distributed in some other way, as far apart from each other as possible. While the well-known spin-statistics connection in particle physics is not a prediction of MOM, this behavior is highly suggestive.

The question of invariance under time reversal is also relevant here. In the discrete-time version of the attraction/repulsion interaction, changing the sign of β is *not* equivalent to time reversal, except in the limit as $\beta \to 0$. However, this limit is the same as the continuous-time version, which gives

$$\frac{dz}{dt} = 2\alpha z(1-z) \text{ , } \alpha = \lim_{\beta\to 0, \Delta t\to 0} \beta/\Delta t \text{ .} \tag{58}$$

In this case we might be able to say, that *reversing time* is equivalent to *exchanging bosonic and fermionic properties of particles*, noting that this is not to be taken as special-relativistic Minkowski time, but as absolute Newtonean time.

In the case of many states(more than two) in the same Hilbert space, it is difficult to make exact predictions beyond the above. The object of interest is the collective, effective density matrix:

$$\hat{D} = (1/N)\sum_{i=1}^{N} |\psi_i\rangle\langle\psi_i| \text{ .} \tag{59}$$

The actual form of the attraction/repulsion interaction, in the case of more than two states, is not uniquely determined so far. Nevertheless, we can deduce some of the features of the evolution of $\hat{D}$, with some plausible assumptions. If the attraction is acomplished pair-wise, then it is possible for any $\hat{D}$ to be constant in time, as long as the states $|\psi_i\rangle$ are all pair-wise either orthogonal or identical. If the attraction is acomplished by $\hat{D}$ acting on each state, then $\hat{D}$ will be constant if either a)all the states are eigenstates of $\hat{D}$, or b)in the diagonal basis, all the elements of $\hat{D}$ are either zero(say, $A - n$ of them, where A is the dimension of the Hilbert space) or a fixed constant equal to $1/n$. Finally, in any version, if $\hat{D}$ is a pure state, it will remain so forever. The stability of these points depends on whether we have attraction or repulsion. It also depends on the exact distribution of states in most cases, as the motion is not 'autonomous.' A pure-state $\hat{D}$ will be stable in the attracting case, unstable in the repulsing case. In general, a partly or totally trivial $\hat{D}$, as in case b) above, will be unstable in the attracting case, stable in the repulsing case. If we make the boson-fermion/attraction-repulsion analogy, this means bosons tend toward all being in a common state, while fermions tend toward all being in orthogonal states. This is consistent with the usual behavior of bosons and fermions, but a more definite prescription would be needed to take the correspondence further.

Conclusion

> . . . One is tempted to suspect that the authors do not understand the Bohr philosophy sufficiently to find it helpful. Einstein himself had great difficulty in reaching a sharp formulation of Bohr's meaning. What hope then for the rest of us?. . . While the founding fathers agonized on the question 'particle' *or* 'wave' deBroglie in 1925 proposed the obvious answer 'particle' *and* 'wave.' *J. S. Bell* [1]
>
> If we have to go on with these damned quantum jumps, then I'm sorry that I ever got involved. *E. Schroedinger* [10]
>
> Our actual situation in science is that we *do* use the classical concepts for the description of the experiments, and it was the problem of quantum theory to find theoretical interpretation of the experiments on this basis. There is no use in discussing what could be done if we were other beings than we are. *W. Heisenberg* [11, p.56]
>
> God may be subtle, but he's not malicious. . . God does not play dice. *Albert Einstein*

All things near and far
hiddenly linked are.
Thou canst not stir a flower
without the troubling of a star.
William Blake

10 General Principles

Let us consider MOM with some general physical principles in mind, beginning with the notion of **observability.** Every observable is a Hermitian operator, but are all Hermitian operators observable? A 'yes' answer reduces, but does not eliminate, the need to explain the measurement process: A 'no' answer begs for a mechanism of some kind to explain the distinction. For example, Bell's **beables**[12] are the variables in quantum field theory given objective reality, in the spirit of DeBroglie-Bohm. In MOM also, not all operators have definite values. The measurement process requires **environmental decoherence**[13, 14, 15]. The interactions between a local system and its environment can be so entangling that no pure quantum state can be ascribed to the local system, but only a reduced density matrix which loses coherence rapidly. It is often taken to represent a probabilistic mixture of orthogonal, observable states. By themselves, these interactions which correlate system and environment are not *sufficient*, without new dynamics, to precipitate measurement outcomes, but they are *necesary* within MOM. In addition, they must exceed a certain energy scale defined by the cyclic time Δt, and they must cross some observer domain cut. Not only do

they decohere the global state, they also define the chaotic dynamics of the *pure* local state. The **pointer basis** or decohering basis is important because it defines the observable to be measured. It is determined, if it exists, by the choice of domains for observers and by the full Hamiltonian. Are **state-vector collapse** and **quantum jumps** real? MOM was designed to produce collapse of the state vector. Allowing for intermittency effects, the system does jump from one classically-allowed state to another, hence these are actual physical phenomena.

Another key issue is **predictability**. Given the statistical nature of the quantum theory, it should not be easy to predict the outcome of a single measurement. Hence the model must be **chaotic**. The dynamics of MOM make it very hard or impossible to predict individual trial outcomes. The required information lies in the fine details of the initial state(s), only approximately known. **Uncertainty relations** are important predictions, and explanatory devices of the quantum theory, so must be respected. Except for time-energy, all the usual uncertainty relations are satisfied by MOM because the world is described using a Hilbert-space formalism. Time-energy itself is satisfied by the specific dynamics of MOM. In discussion of quantum measurements, the language of **counter-factuals**[16] is often used. Stapp's paper offers an 'alternative approach'(p.14) assuming that a definite outcome to a measurement procedure exists *if* it is carried out: If another observable is measured, no counter-factual assumption about a definite outcome need be made. (I.e. What would be the result *if* the original measurement *had* been made.) This applies even if the two observables commute. MOM obeys Stapp's assumption because every distinct measurement procedure involves its own Hamiltonian and unique chaotic evolution, even when commuting operators(observables) are involved. This enables MOM to avoid various EPR-type **no-go theorems.**

There are other issues of interest. **Statistics:** The exchange symmetry of identical particles may be violated by a naive version of MOM. Extension to quantum fields, defined over finite regions of space, should solve this problem. **Relativity:** MOM assumes the existence of a universal cyclic time interval governing all quantum evolution. This is Newtonean, rather than relativistic time, and might be defined by the rest frame of the cosmic background radiation. **Non-locality:** In MOM, the global state vector maintains non-local correlations until a measurement outcome is selected. If this state is absent, however, correlations might fail over sufficiently large distances. **Time asymmetry:** [17] Zeh's paper and others suggest that the quantum measurement process is the origin of time asymmetry in physics. SE itself is time-symmetric, so one expects it cannot decribe all aspects of the process. MOM incorporates time asymmetry directly via chaotic dynamical maps, which are non-invertible: No continuous-time dynamics can have this property. Time reversal is explicitly broken only in the chaotic regime, i.e. during a measurement. Between measurements, the evolution is time-symmetric, closely approximating SE.

11 Specific Models

Here different models and viewpoints are discussed, with emphasis on similarities and differences from MOM.

Orthodox viewpoints(*'The tried and true.'*)

These are not really models of measurement, but rather explanations and prescriptions rooted in the early triumphs of the theory, namely the probabilistic rule(BP) connecting theoretical quantities to experiment. They are the most widely accepted due to their apparent validity and complete lack of speculative content.

The Copenhagen School(Bohr): This school of thought is well-known. The necesity to describe the measuring apparatus in classical language is transformed, in MOM, into the assumption of the existence of the cut, that certain systems(observers) have their own state-vectors, in addition to the global quantum state. The *unanalizability* of the measurement process is denied by MOM: Instead definite dynamics are introduced. The fundamental role of system-observer interactions in the measurement process is reasserted.

Statistical interpretation(stat): This is probably the most widely-held view of quantum theory. The quantum state *does not* represent a single microscopic system, rather an ensemble. The probabilities defined by the Born rule are not very different from those defined classically, but *something* about small systems prevents a more precise description, available for larger, classical systems. In MOM also the probabilities are statistical, but there is additional information, extra quantum states which determine individual measurement outcomes.

Decohering histories(Dec): In this picture, the quantum state of the universe is interpreted as the *incoherent* sum of a set of 'histories' of the world. Each history is a set of possible measurement outcomes, for observables at different times. These can be time-dependent, and they answer the question, what observables have well-defined values, and when? The set of decohering histories may not exist, as there is a set of logical requirements they must satisfy, namely that the quantum state can be treated *as if* the universe were in a *classical* probabilistic mixture of these histories. The question remains, whether in reality only one of the histories is factual, or instead they all somehow exist in parallel. MOM singles out a *unique* result to every measurement.

Reactionary models(*'We've gone too far...'*)

Some, fearing that the quantum theory introduces too much uncertainty into nature, seek to re-solidify it by asserting the definite existence of *something*. The most basic instinct is to give reality to classical variables, but this approach can be much more subtle.

DeBroglie-Bohm model(DB):[18, 19, 20] They postulate that in addition to the wavefunction there are classical variables which evolve deterministically or stochastically, giving objective reality to the world. The quantum state is not affected by them, but evolves by SE, while the classical variables respect the quantum probability distribution. MOM also adds information(extra quantum states) which helps to define a more objective reality for some variables, but in an indirect way: The interaction between quantum states induces measurements at well-defined times. In MOM, however, objective reality is ascribed to quantum states, not classical variables.

Many-worlds(MW): In this picture of reality, there is a universe for every sequence of possible outcomes, for all measurements ever performed. Each universe embodies a unique outcome for each measurement, and does not interact with the other, 'parallel' universes. This raises a question about the *role* of the observer: Does it split up into many observers after each measurement? The meaning of probability becomes less clear. The MW viewpoint dispenses with collapse, but postulates innummerable universes, each equally *real*, logically separated from each other. Also, the observables and splitting times are not specified. MOM is quite different, as it seeks to describe a *single* universe with *many* observers, rather than *many* universes each with a *single* observer.

Modal interpretation(mod): This approach hopes, that by analysis of the quantum state, including all the environment degrees of freedom, it will be possible to prove, that the SE leads to collapse of the wavefunction. Thus, the state makes a choice among the possible outcomes, based on information contained in the environment. The observables of the theory should arise *spontaneously* from such behavior, and BP should be recovered. There is no evidence for this, but hope springs eternal. The information deciding measurement outcomes must come from somewhere, so the environment is a plausible candidate. The success of such a program, however, would deny the applicability of the standard interpretation to the wavefunction of the universe. MOM assumes from the beginning, that this *will not work*: The information must come from somewhere other than the single state of the standard quantum theory.

Progressive models(*'Onward and upward!'*)

Others feel the quantum theory is *incomplete*: It needs a broader conceptual scheme, or at least new dynamics. This is not a return to classical ways, rather speculation on the more *radical* features of the theory, hoping they will point us in the right direction, towards new physics.

Conscious observer(Wigner):[21, 22] In this picture, a measurement outcome results when the mind of a conscious observer becomes correlated with the quantum system. It is impossible for such a mind to be in a state of fundamental uncertainty for long, so instead, an outcome is selected. The conscious observer becomes a *participator* in the process of reality-building. MOM *allows* for each state-vector to be associated with an independent conscious observer, but this is not necesary, and has no role in the dynamics. However, I favor that interpretation. If true, this means that consciousness is involved in the resolution of uncertainties above some energy scale. At some point, the distinction between the quantum and the classical worlds comes about: Which variables resist indeterminancy, entanglement with their environment? MOM is a way to explore this issue, with or without consciousness.

Penrose: He presents an alternative to either deterministic or stochastic evolution, for state-vector collapse: a *non-algorithmic* deterministic process. I.e., although fully determined, quantum measurement outcomes can never, even in principle, be predicted. In MOM, the chaotic dynamics preclude exact prediction of particular measurement outcomes at present, so the distinction is not yet important. For Penrose, spacetime variables(e.g. metric) resist quantum indeterminism. If he is right, no *naive* implementation of 'quantum gravity' can succeed. Thus, the variables with 'observer status' are spacetime-structure-related: perhaps at some level, the positions of particles, yet at a deeper level, the fabric of spacetime itself. MOM can test these ideas by defining the observer cuts appropriately.

Stochastic models (GRW): [23, 24, 3, 25] [13, Gallis and Fleming] In GRW stochastic dynamics modifies the Schroedinger equation, effectively driving the system to one of a small set of states. These represent the measurement outcomes, and are reached according to BP. MOM also assumes some new dynamics, but is deterministic and involves multiple quantum states. In the standard measurement case, the following picture is useful: The global state undergoes a pseudo-random walk in response to the chaotic motion of the local state, which culminates in a measurement outcome. This is similar to GRW, but uses chaotic determinism in place of randomness.

Many-minds(Albert):[26] This work is an effort to improve upon the many-worlds interpretation by incorporating the conscious-observer postulate. I.e., there *are* multiple universes, wherein measurements have achieved different results, and the mind of the observer *does* gets split at each measurement(really!). There is a different mind for each universe, and consciousness is the reason for the measurement process. Yet there is no collapse. An attempt is made to place consciousness at center stage, rather than classical variables, hence this is a progressive view, unlike MW. MOM assumes that it is *not enough* to recognize observers have a role. One must also introduce *specific* new dynamics into the theory to go beyond orthodoxy in a useful way.

12 Quantum Measurement Categories

Here the various schemes discussed above are categorized using some obvious criteria, and the results are shown in Figure 4. But first we reject the **bare theory without collapse**: This is the null interpretation, without deviation from SE. It cannot be considered an adequate model of reality, unless measurements follow strictly from SE. But this is not so, hence some additional structure is needed. Although it has some adherents(also see modal interpretation), it leaves little room for discussion.

Observables

Are there any classical observables in the theory? These are functions of the classical dynamical variables with well-defined values at some or all times, when interpreted as quantum operators. If so, are they fixed or do they depend on the state of the system or its interactions?

Space-time(position, gravity): (*GRW, DB, Penrose*) The observables are associated with the structure of spacetime. Either particles' positions (GRW, DB), or the curvature of space (Penrose) receive special status: They have well-defined values or resist uncertainty.

Knowledge/mind: (*Wigner, Albert*) There exists an agent external to the ordinary degrees of freedom of the quantum state: the conscious mind. This affects the evolution of the system by choosing from the various possible measurement outcomes (Wigner). Or it splits into many minds whenever a measurement takes place, giving reality to all possibilities (Albert).

Defined by classical apparatus and interactions: (*Bohr, stat, MW*) According to Bohr and most physicists, the observable is determined by the arrangement of the classical apparatus and its interactions with the microsystem. This at once opens possibilities and constrains them: Any operator may be an observable, but only if a suitable experimental arrangement can be constructed.

Defined by the quantum state: (*Dec, mod*) These models rely on the SE evolution and analysis of the state to determine observables. Pointer-basis states or decohering histories are assigned probabilities which are interpreted classically.

Defined by multiple states and interactions: (*MOM*) The observables are defined by the choice of cuts and the interactions across them. Thus no specific observable is selected a priori.

Number of Outcomes

Does each measurement have a unique outcome, or does anything that could happen actually happen, in a set of 'parallel universes'?

One, determined causally: (*DB, MOM*) In DB a single choice is made for the initial positions of every particle, which determines the outcome of every subsequent measurement. In MOM there is also a set of initial data which determines the future evolution, including measurement outcomes, but this takes the form of quantum states, not classical variables.

One, non-algorithmic: (*Penrose*) He proposes that the outcome of a given measurement is determined, but in a non-algorithmic way. Hence prediction is impossible in principle. The difference between this and random choice may be only of philosophical interest.

One, random: (*Bohr, stat, Wigner, GRW, Dec*) There is an outcome to each measurement, but it occurs through a fundamentally random process, whether for a single event or the history of the universe. The reality of the universe, within a quantum description, is explained by appeal to mysterious influences beyond scientific investigation.

All possible: (*MW, Albert*) We give up the notion that there is a single outcome to a given measurement: Instead, all possible outcomes actually happen. Does mind only follow one path, leaving the rest barren of consciousness, or does it split up among the paths, so they all acquire reality? The possible existence of parallel universes forces us to consider the role of mind in quantum theory.

Process

This is usually ignored by most interpreters of quantum theory, but otherwise there are only two options.

Random: (*GRW*) They use a random Brownian-like motion to nudge the state into one or another measurement outcome. It works by replacing a large random effect by the concatenation of many small random effects. Should one go further, and explain the small random effects by introducing yet smaller random effects? I don't think this approach is promising. Rather, one should recognize randomness in a

physical theory as an indication of a *temporary* lack of understanding of the processes involved. We should not solve one mystery with another.

Deterministic: (*DB, MOM*) The basic DB model is entirely deterministic for both the observable and the quantum state, but does not allow easy prediction of future measurement outcomes. MOM explicitly includes chaotic deterministic evolution, so predictions are nearly impossible. However, both models offer at least some possibility of *prediction* and *control* of quantum events.

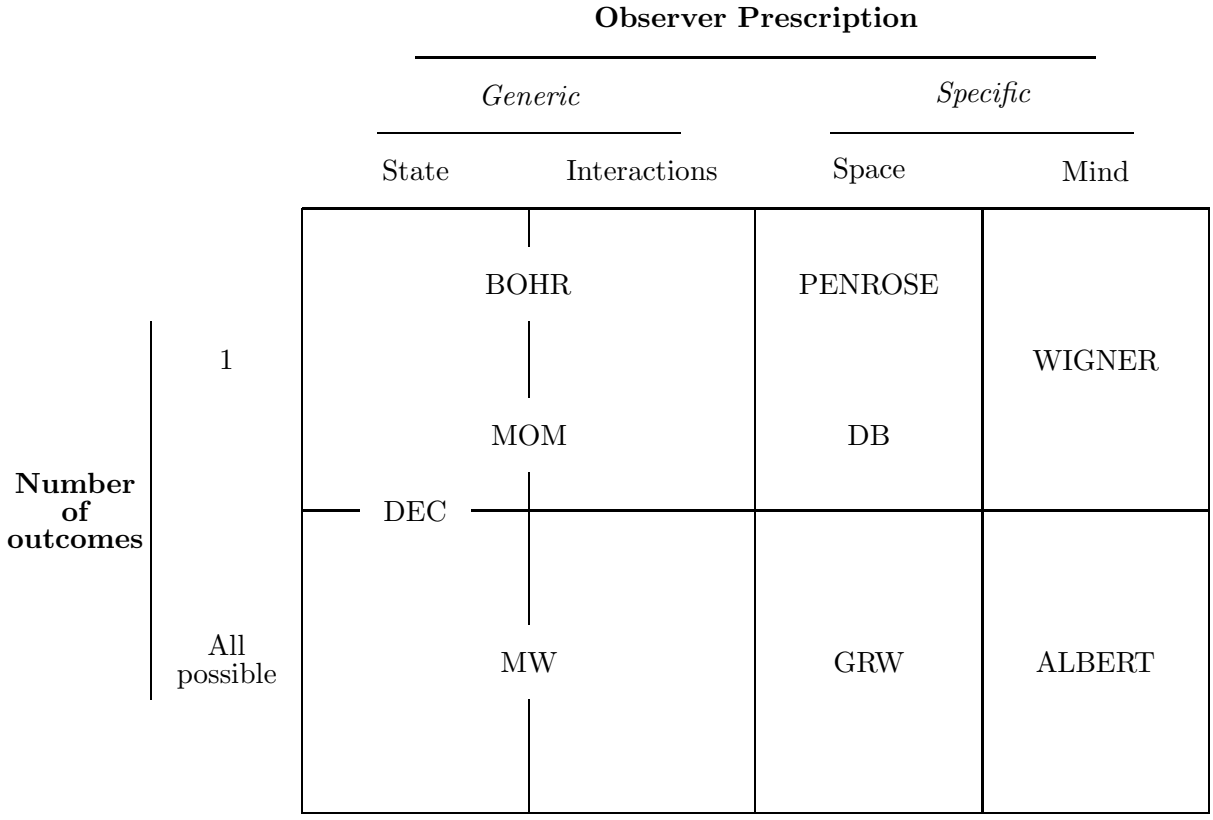


Figure 4: Quantum measurement classification, indicating the choices made by different schemes. In this analysis, MOM is closer in substance to Bohr than any of the others, notwithstanding the philosophical differences between the two.

13 Outlook

Although MOM has been restricted above to simple applications, it is a general formalism. However, it is somewhat *ad hoc* and hence not yet a complete *quantum theory of observers*. It seems likely that a more fundamental quantum-type theory of nature underlies MOM. Let us consider the parameters and other arbitrary aspects of MOM, and how they might be constrained.

Cyclic(discrete) time Δt: MOM assumes discrete-time evolution of nature from the start. The underlying theory could either provide a continuum description at smaller time scales, or prove the impossibility of using continuous time, given certain constraints on the evolution. The actual value of Δt might be found to be the Planck time, or a different time scale, or even observer-dependent. Within MOM it remains an arbitrary constant, assumed for now universal.

Decoherence map: The decoherence map itself is somewhat mysterious. It is introduced to make the dynamics chaotic, which is crucial to achieve the desired results. In a better theory of the time cycle, this could be a kind of recurrence map. Perhaps the local state is well defined only at specific times, whereas usually it is only potential, inherent in a larger structure. A more complicated object would be sometimes a member of one Hilbert space or another, but most times something more rich in information than any state. This could be a new kind of 'super-state', or a general density matrix on the global Hilbert space.

Universal interaction of observers: This attraction is also new, used to enable the different observers to agree on measurement outcomes. Because it involves pairs of states, possibly in different Hilbert spaces, the need for a 'super-state' formulation is even greater here. If two objects interact in some way, then they must somehow be *very* similar, probably just different individuals of the same 'species'. The interaction strengths β and γ are arbitrary, but there is indication that they depend on the dimensionality of the state spaces. The strength should be near maximum($\gamma \approx 1$) for a local state in a large environment, but very small($\beta \ll 1$) for the global state when the local degrees of freedom are few. It may be possible to constrain β from below, if collapse of the wavefunction is real.

Number, placement and dynamics of cuts: For measurements to occur, the state spaces and the Hamiltonian must be right. Interactions must operate across an 'active' cut, and the decoherence time must be much shorter than Δt. But given a reasonably-sized universal set $\mathcal{U}$ of classical degrees of freedom, the set of all possible cuts is very large: the power set of $\mathcal{U}$. Conceivably, every one of the associated Hilbert spaces could be populated by any number of quantum states. If each cut has even *one* state, that is a lot of states. Do we live in such a universe? If not, can the active ones 'move', their domains changing with time according to some rule? For now, MOM simply assumes the active cuts are few in number, their domains fixed.

Wavefunction of the universe: Whether or not this exists is relevant to quantum cosmology. The number of observers/participators/state-vectors could be very large. It is conceivable that there is no 'wavefunction of the universe', but that instead there is an effective collective state(density matrix) with contributions from all observers. Would the underlying theory *require* the existence of the wavefunction of the universe, associated with an active cut whose domain contains all other domains? So far, MOM assumes a global state exists.

Multiple states in same Hilbert space: If there are many state vectors in the same space, would this lead to different measurement behavior? Their existence could make it seem as though a quantum field were present: They would act like a set of identical particles. If the underlying theory prescribed which cuts are 'multiply activated', then it would predict what are the 'fundamental quantum fields.'

Let us now consider what kind of measurement result would indicate the need for a quantum theory of observers: a detailed account of how measurements actually occur. All measurements involving micro-systems have so far been of the *simple* type. The system of interest, including the micro-system and some apparatus, is isolated as well as possible, then it is allowed to evolve for some time, and finally a standard measurement is carried out on it, using additional apparatus. The initial state of the system may be exactly known, or a statistical mixture may be used to describe it, but once this initial information is defined, it evolves strictly by SE. The results of the measurement, at the end of the isolated evolution, can be checked against the predictions of SE and BP. They have always turned out to agree, within statistical uncertainty. This includes all normal measurements, as well as those checking 'strange' quantum effects such as the Aharonov-Bohm effect, or EPR-type correlations. If experimental results are *incompatible* with BP, then there are three possibilities: a) The theoretical description of the system and its evolution is inadequate, because important interactions have been ignored, or relevant degrees of freedom left out; b)

the predictions of the standard quantum theory, namely BP, are being violated in a fundamental way; or c) in between the preparation phase, which defined the initial state, and the final measurement phase, there was an unexpected, spontaneously-occurring measurement of some kind. A type (c) result will be called a *compound* measurement, because it involves additional measurement processes between the preparation phase and the final state. Note that this is distinct from the successive carrying-out of more than one simple measurement. So far, it has always been possible to trace the problem to option (a). Thus, there is no evidence for either fundamental deviations from BP, or spontaneously-occurring measurements: All measurements are *caused* by explicit actions of the experimenter, and the results *always* agree with BP. All well-formulated models or theories about the measurement process, such as collapse models like MOM, must wait for such new experimental results, before they can be judged. These new experiments must form the *foundation* for the *possibility* of studying the measurement process itself. Perhaps some compound measurements have already been carried out. These involve 'intermittent' or 'continuous' measurements, with Hamiltonians which are not of the pointer-basis type, but contain kinetic as well as decohering terms.

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