

D:(30 pts) Incline Masses and Pulley

A block with mass $m_1 = 90\text{ kg}$ sits on a rough incline with angle $\theta = 20^\circ$. It is connected by a string over a pulley to a hanging mass. The incline surface has coefficient of static friction $\mu_s = 0.4$ and coefficient of kinetic friction $\mu_k = 0.35$. What is the maximum value of the hanging mass m_2 that will keep the system in equilibrium?

•(15pts) $m_{2,\text{max}} = \underline{64.6 \text{ kg}}$

Static limit: $a=0$, $f = f_{s,\text{max}}$
 $f = \mu_s N = \mu_s m_1 g \cos \theta$

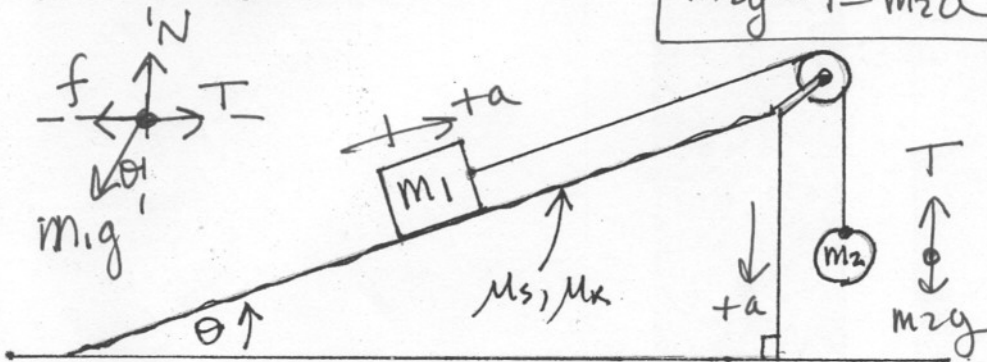
$\textcircled{I} + \textcircled{II} \Rightarrow m_2 g - m_1 g \sin \theta - f = 0$

$m_2 g = m_1 g \sin \theta + \mu_s m_1 g \cos \theta$

$m_2 = m_1 (\sin \theta + \mu_s \cos \theta)$
 $= \underline{64.6107 \text{ kg}}$
 $\approx \underline{64.6 \text{ kg}}$

$T - m_1 g \sin \theta - f = m_1 a$ \textcircled{I}
 $N = m_1 g \cos \theta$

$m_2 g - T = m_2 a$ \textcircled{II}



If instead, the value of the hanging mass is $m_2 = 80\text{ kg}$, this is too much and the block moves up the incline. What is the value of the acceleration a ?

•(15pts) $a = \underline{1.13 \text{ m/s}^2}$

$a \neq 0$, $f = f_k = \mu_k N = \mu_k m_1 g \cos \theta$

$\textcircled{I} + \textcircled{II} \Rightarrow m_2 g - m_1 g \sin \theta - \mu_k m_1 g \cos \theta = m_1 a + m_2 a$

$g [m_2 - m_1 (\sin \theta + \mu_k \cos \theta)] = (m_1 + m_2) a$

$a = g \cdot \frac{m_2 - m_1 (\sin \theta + \mu_k \cos \theta)}{m_1 + m_2} = \underline{1.13091 \text{ m/s}^2}$
 $\approx \underline{1.13 \text{ m/s}^2}$